Zurich University of Applied Sciences and ETH(*) Contribution to the High-Angle Working Group

(*) With help from ETH students

Marcello Righi, AePW4, June 5, 2026

Benchmark Supercritical Wing

- Simple kinematics (pitch-plunge)
- Complex physics, Shock-Boundary Layer Interaction possibly beyond RANS/URANS
- Uncertainties: Results in previous AePW turned out to be dependent on spatial, temporal resolution, turbulence modelling, solver settings (e.g. limiters)
- Full-order model solution (CFD - structural dynamics) computationally expensive
- ROM may help identifying the role played by the causes of uncertainty and limiting the computational costs
- Volterra Series

Aeroelastic solver

State-space system with linear and nonlinear aerodynamic forces

- Equations of motion:

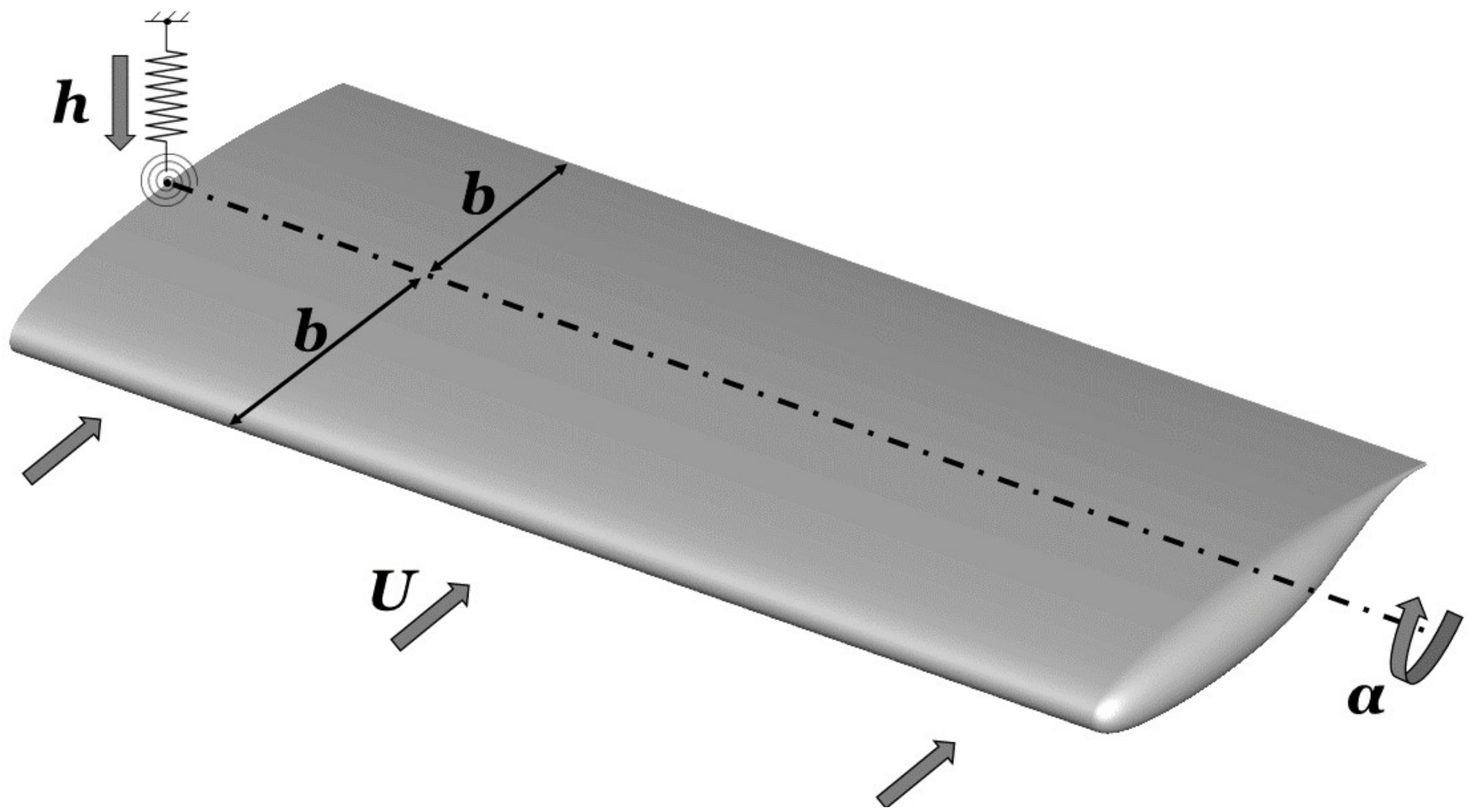
$$M\ddot{x} + C\dot{x} + Kx = f_a(t)$$

- Recast as discrete time state-space system:

$$x^{n+1} = Ax^n + f_a^{\,n}$$

- ROM, Volterra Series (simplified!):

$$f_a^{\,n} \simeq \sum_{i=1}^{m} h_i^1\, x^{n-i+1} + \sum_{i=1}^{m} h_i^2\, \left(x^{n-i+1}\right)^2 + \sum_{i=1}^{m} h_i^3\, \left(x^{n-i+1}\right)^3 + \dots$$



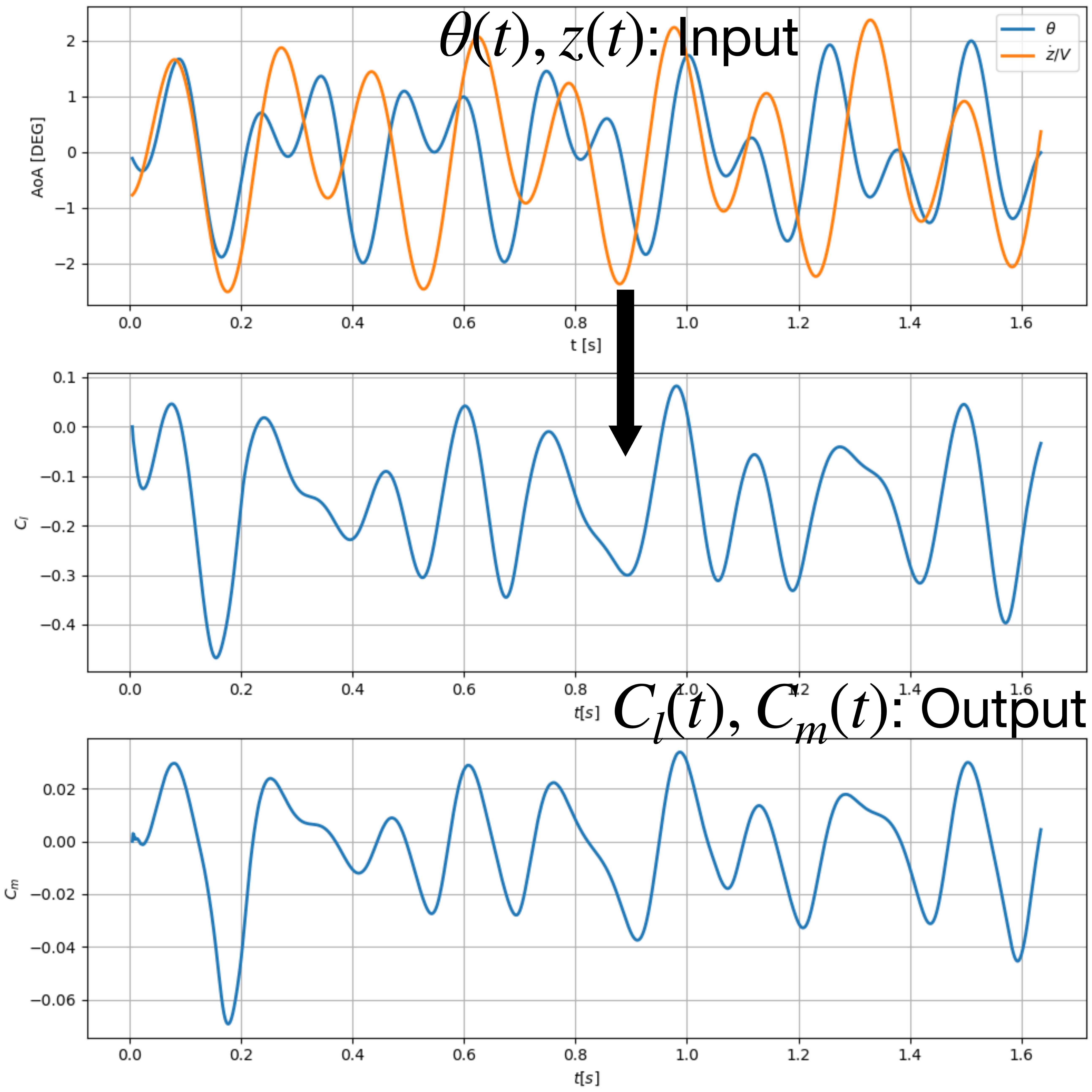
- "Augmented" system to include the linear kernel in the state-space system:

$$\tilde{x}^{n+1} = \tilde{A}\tilde{x}^n + \sum_{i=1}^{m} h_i^2\, \left(\tilde{x}^{n-i+1}\right)^2 + \sum_{i=1}^{m} h_i^3\, \left(\tilde{x}_{n-i+1}\right)^3 + \dots$$

- Analysis of the eigenvalues of the state matrix $\tilde{A}$ and/or time integration

Identification

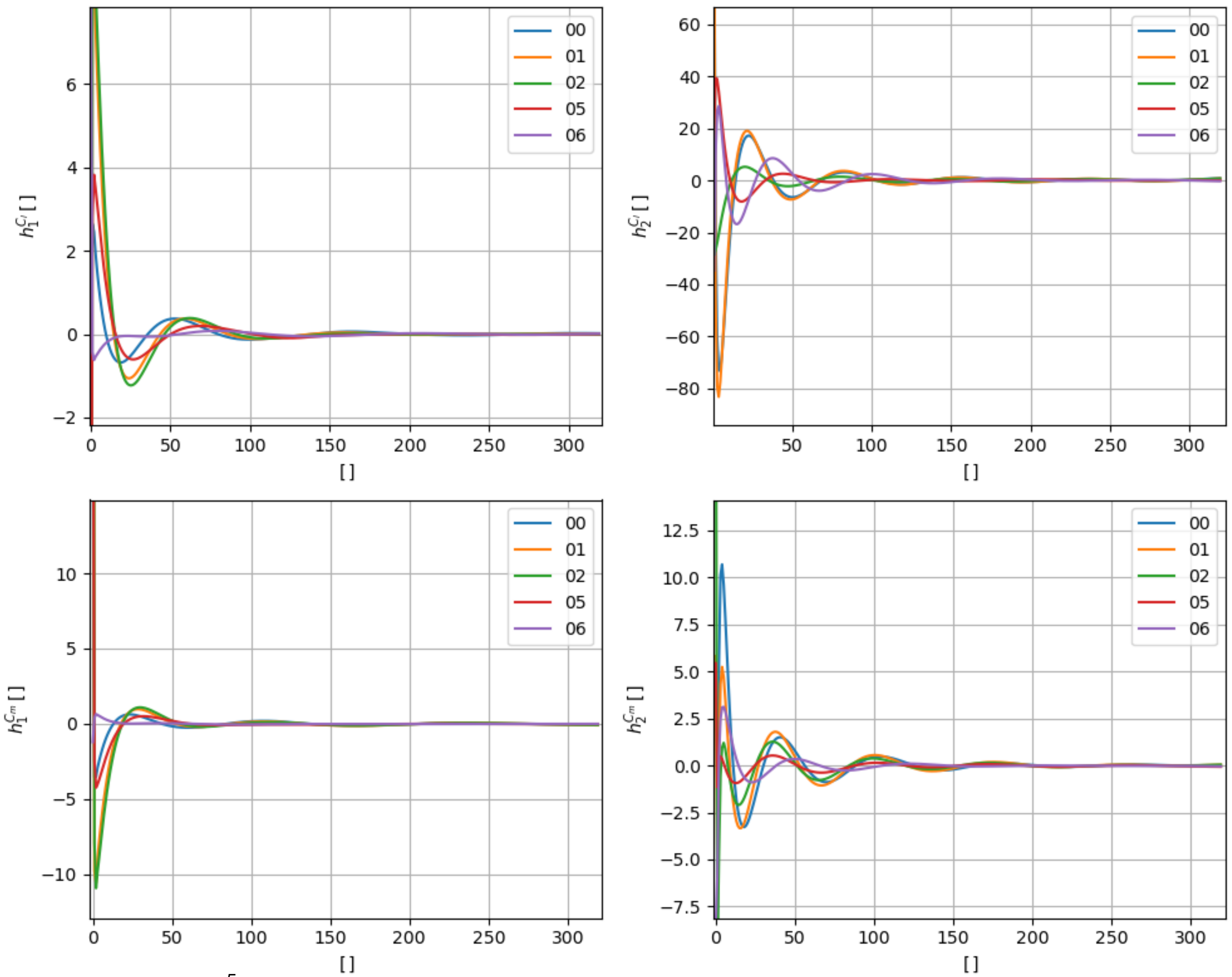
- Band-limited multisine input signals, between 4 Hz and 25 Hz
- Pitch and plunge identified from the same simulation
- Curse-of-dimensionality: it might be limited, as shown by recent studies (2021 onwards). Sequential identification, prescribed sparsity (Earl Dowell, Michael Candon).
- The claim is that Volterra series may “characterise the centre manifold”, i.e. provide some information on the Hopf bifurcation.



Volterra Kernels

$h_i^1, h_i^2, h_i^3 \ldots$

- Kernel coefficients are arrays, size *m*
- Kernels vary significantly with AoA and M but keep similarities.
- Simulations with SU2, very good quality Case 3 grids, and data from Dr Candon RMIT for Case 1.
- Time step *0.00025 s*, Spalart-Allmaras, Volterra memory depth *m = 320*



Volterra Series

Convergence? Validity limits?

- Nonlinear Volterra series may capture mild nonlinearity with sufficient accuracy but when is a Volterra Series consistently identified?

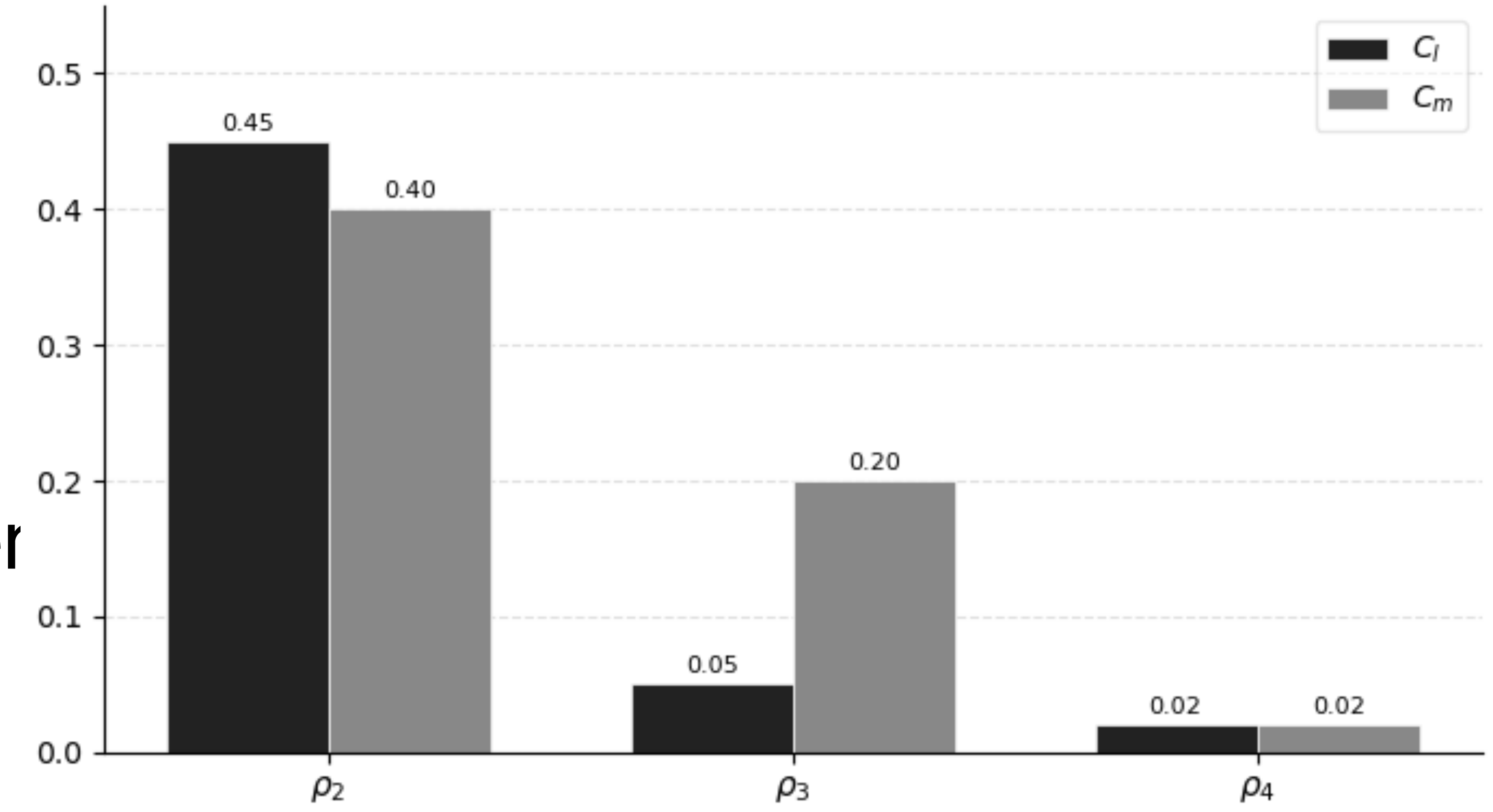
- Range/amplitude of training signals must be larger than response observed in flutter tracking simulations. Is this a sufficient requirement?

- No, we can find additional criteria, among which that the Series must converge, i.e. higher-order kernels are not increasing as order goes up:

$$\rho_n = \frac{A\,H_n}{H_{n-1}} \text{ must be } < 1$$

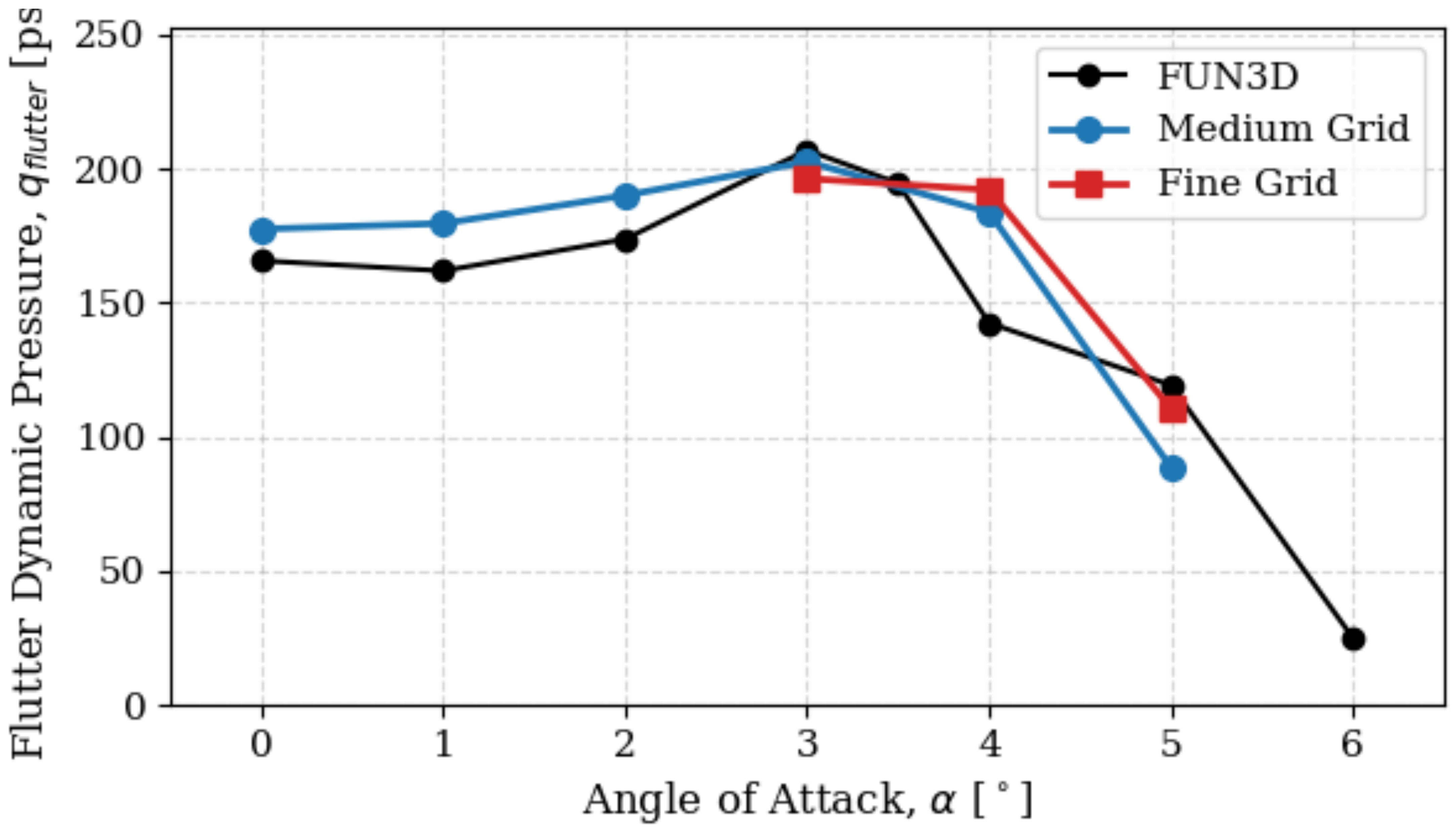
with *A* the amplitude of the signal. In the present implem

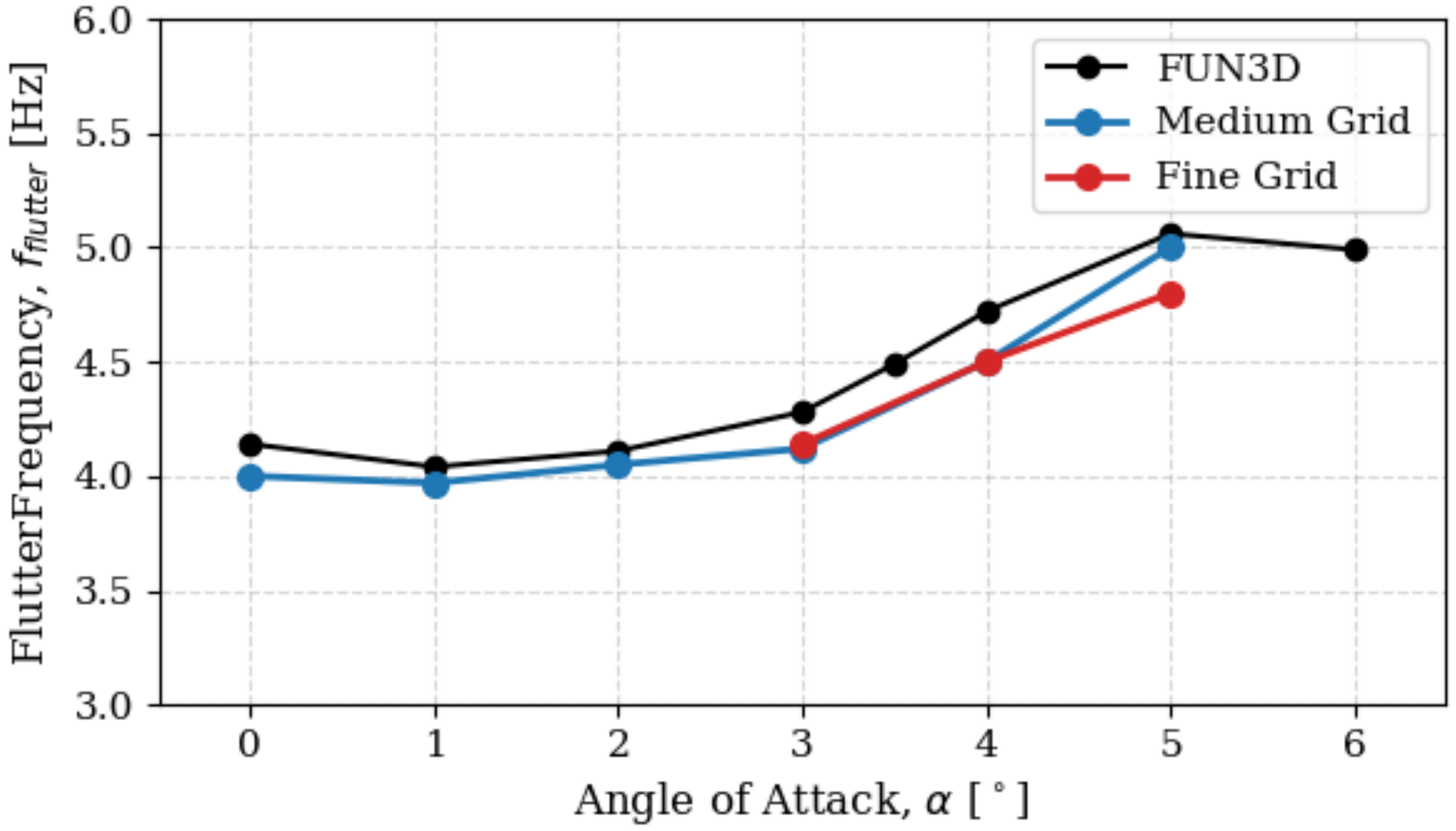
$$\rho_1 < 0.5, \rho_2 < 0.3, \rho_3 < 0.05$$



Case 1, AoA Sweep, M=0.80

- change of dynamic behaviour between the AoA ranges $\alpha \leq 3°, \quad \alpha > 3°$, linear kernels dominates the former case (eigenvalue analysis is sufficient), higher-order kernels dominate the second case (LCO appear even if the state matrix is “stable”).
- flutter frequency and mode move from 2 DoFs to 1 DoF as qf reduces (in general flutter frequency seems to be inversely proportional to flutter q).
- We may infer a subcritical LCO in the second AoA range, with LCO appearing before flutter point.

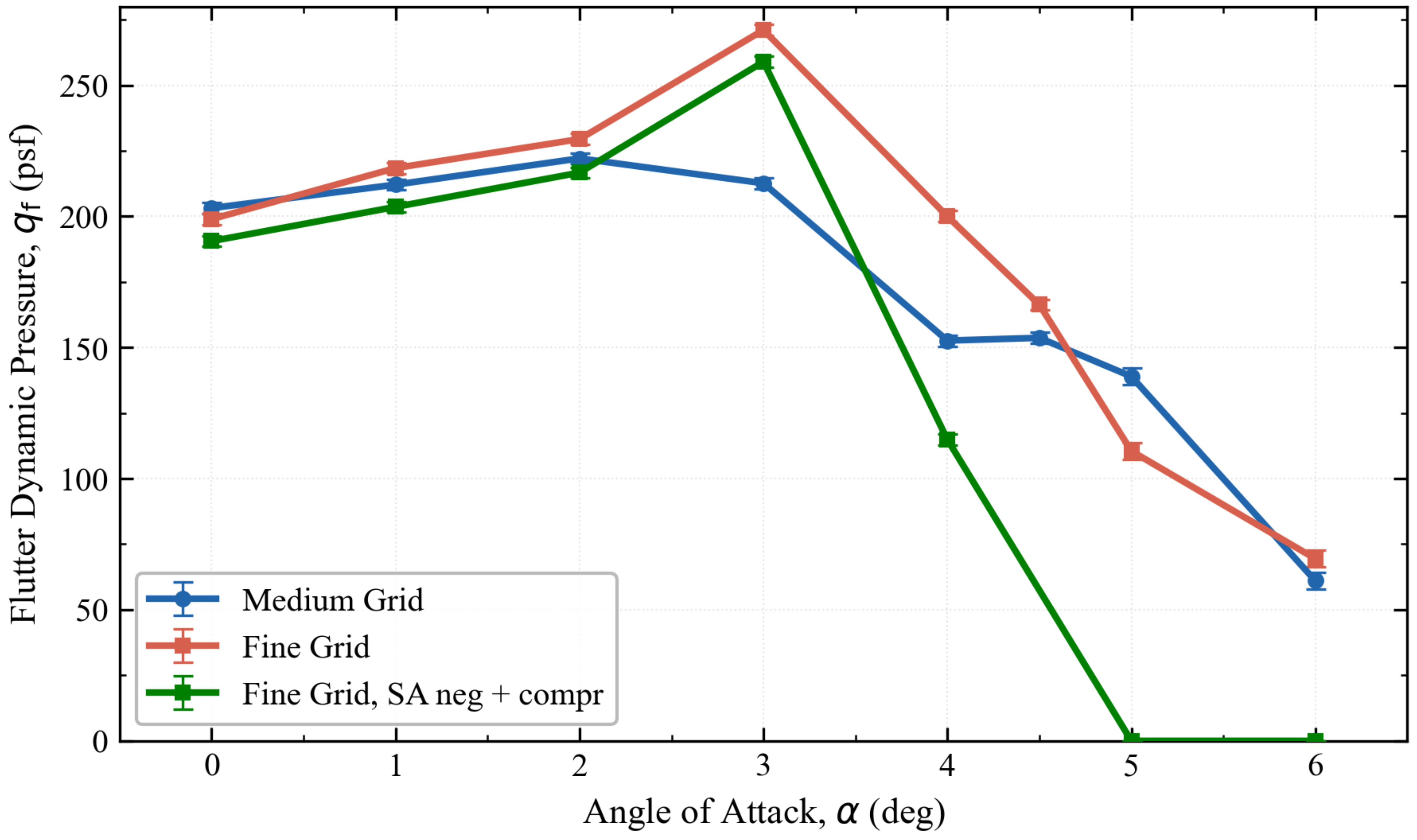


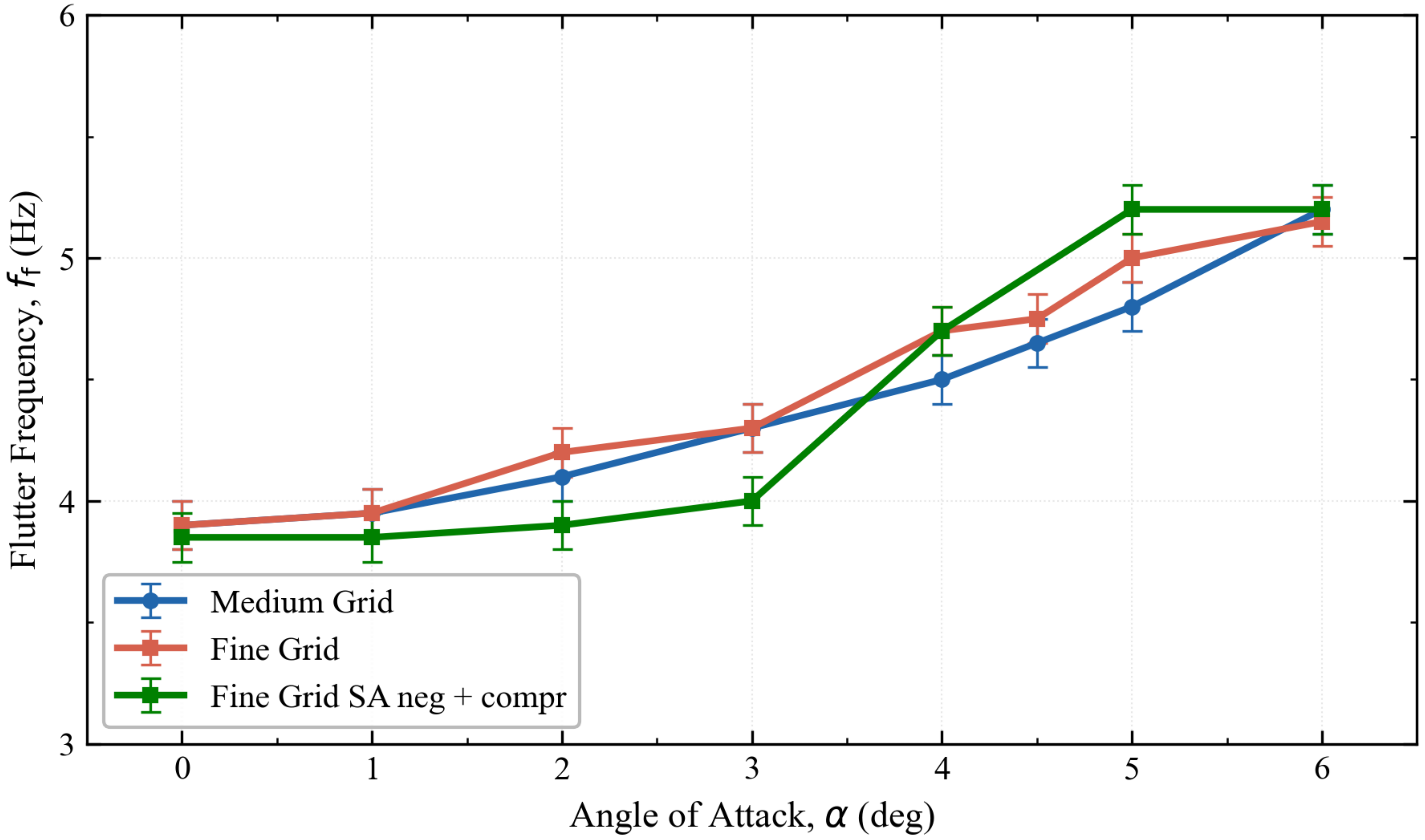


FUN3D data from B. Stanford, P. Chwalowski, K. Jacobson, TRANSONIC LIMIT CYCLE OSCILLATIONS OF THE BENCHMARK SUPERCRITICAL WING, IFASD 2024

Case 3, 2D, AoA Sweep, M=0.80

- Flutter (q, f) plausible, comparable with FUN3D results published by HAWG.
- Similar patterns noted in Case 1, with an abrupt transition to single DoF flutter with SA options active.
- Spatial resolution also plays a role.
- Spatial resolution, insufficient CFD convergence and physical nonlinearity affect identification.





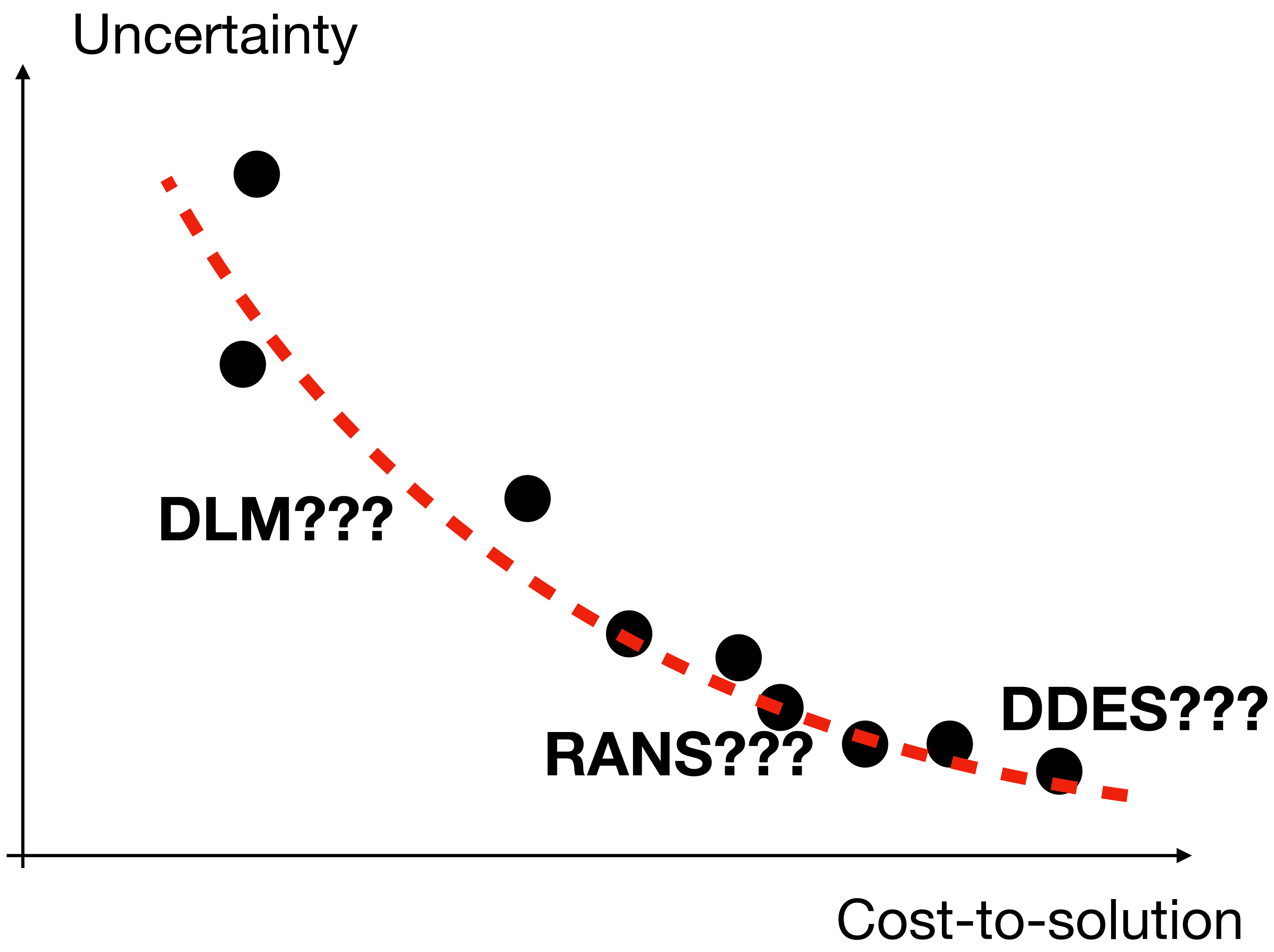
Conclusions

- Flutter data plausible and in line with FUN3D despite substantially lower computational costs (although FUN3D results very valuable for setting up!), transition from 2 DoFs to 1 DoF flutter captured
- Linear instability captured sharply (eigenvalues) at lower AoA, at higher AoA higher-order kernels dominate the dynamic response, subcritical LCO, can Volterra characterise the centre-manifold?
- Data spatial and temporal resolution to be investigated
- Volterra kernels identification may be further improved / training costs may be further reduced
- Flutter tracking approach (perturbation, static aeroelastic solution, multiple estimates of Volterra kernels etc)
- Uncertainty due to RANS turbulence modelling may be more accurately quantified

Future Directions

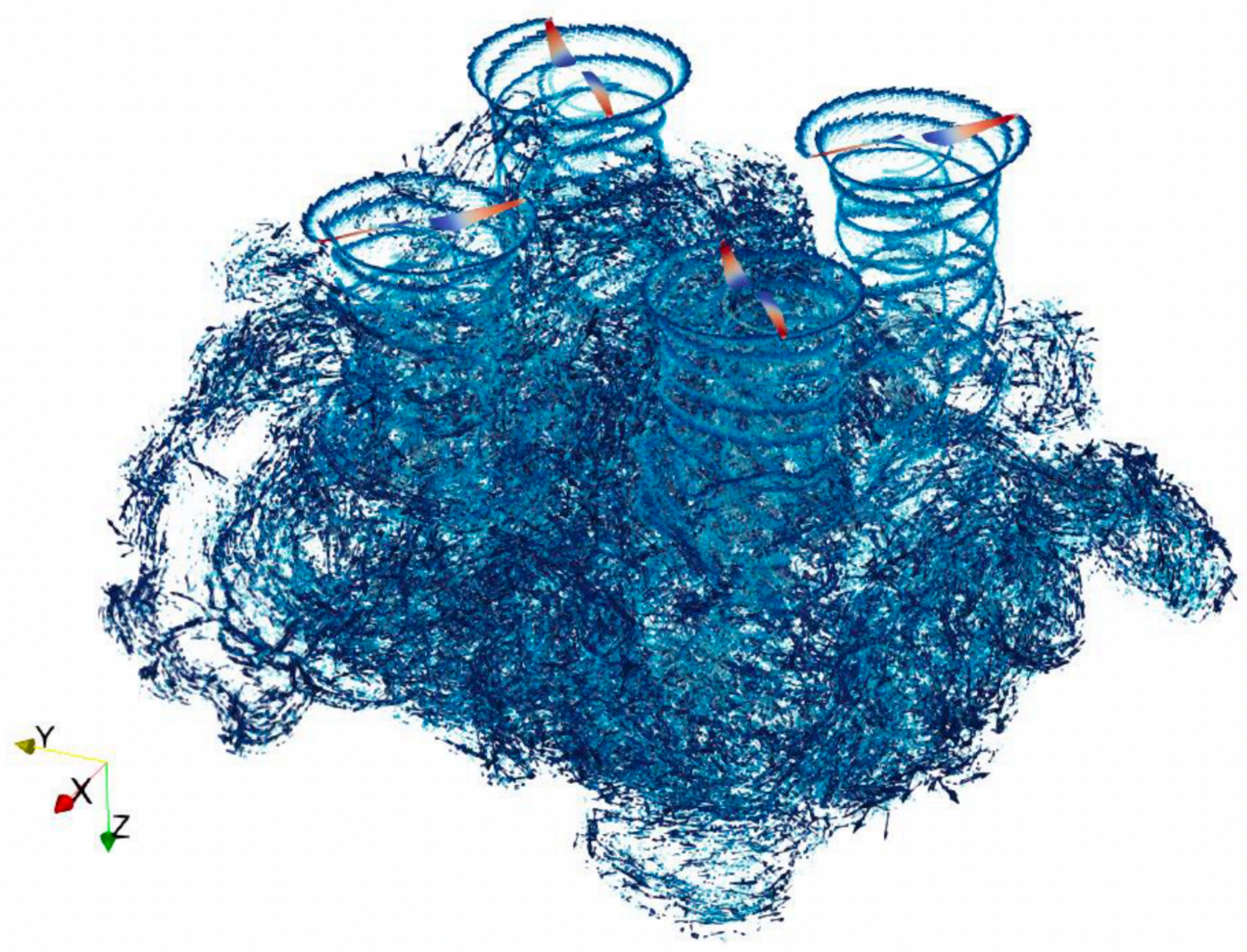
AePW4 HAWG: Possible Extensions: BSCW and Parameter Space Studies

- Assess the static aeroelastic response, flutter dynamic pressure, and flutter frequency over a parameter space broader than that explored in AePW4. The objective is to force the participants to also consider “cheaper” options.
- Plot expected uncertainty against computational cost-to-solution on a chart.
- Investigate data fusion / multifidelity approaches incorporating experimental / WT data. Making CFD goal interpolating between experimental points. The exercise may also include the assessment of experimental uncertainty.



Future Directions

Additional Physical Phenomena



- **Limit Cycle Oscillations (LCO):** Numerically characterize a subcritical Hopf bifurcation for a selected test case (BSCW, Pazy, or another suitable configuration). The exercise is very challenging as it requires accurate nonlinear terms (similarly to BSCW at M=0.8, AOA> 4.5 where the Jacobian determinant vanishes).

- **Large Deflections and Rotor Dynamics with Mid-Fidelity Solvers:** Characterize large-deflection and rotor dynamic effects using mid-fidelity solvers such as DUST or FlowUnsteady. While not particularly novel, this topic is of growing industrial interest, particularly for assessing propeller effects including whirl flutter. Mid-fidelity solvers are computationally efficient and model vorticity accurately, although they ability to capture surface pressure distributions is limited (at best). The test case should therefore be selected carefully to ensure a consistent RANS / mid-fidelity comparison: conditions should exclude boundary layer separation and compressibility effects, with lift dominated by tip vortices. Candidate test cases include rotor flows and possibly the Pazy's LCO.

- **Freeplay:** One possible exercise would prescribe a freeplay condition and invite participants to incorporate it into their aeroelastic solvers. Alternatively, the exercise could be built around an existing (or planned) experiment. Refs. by Cristina Riso for examples.

Future Directions

Industry

- **Limit Cycle Oscillations (LCO):** Numerically characterize a subcritical Hopf bifurcation for a “realistic” test case. The idea specifically addresses “benign” LCOs, (the effects of which) EASA and FAA are so keen to quantify.
- **Control Surface Hinge Moments:** Many aeroelasticians in industry are highly interested in control moments yet remain cautious about CFD-based assessments of these quantities. A challenging test case could focus on the hinge moment of a realistic three-dimensional control surface, possibly under failure conditions, specifically in the presence of pressure leaks between the two sides of the control surface.
- **Freeplay as uncertain quantity:** “When you measure it, free play is never what you expect it to be!”