Overview of Mathematical Modeling Methods for the AFRL RC-19 Configuration

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Summary

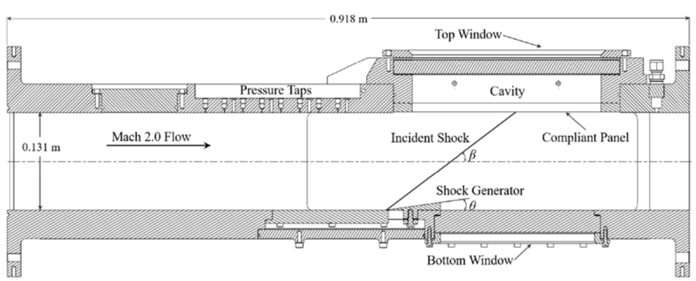
- A range of aerodynamic models has been considered including piston theory, full potential flow, Euler flow and RANS with and without shock impingement.
- A **finite element model** of the plate and its support structure allows the **determination of the effective in-plane support boundary condition.** This information could also be obtained from an experiment to measure the change in natural frequencies due to a pressure differential.
- For the RC-19 configuration the **results are particularly sensitive to the pressure differential, thermal stress** (which leads to buckling) **and the in-plane as well as out of plane boundary support conditions** for the plate.
- **Results for flutter and LCO of the RC-19 experiment are not particularly sensitive to the aerodynamic model** (**thin BL →** Expected small boundary layer effects), with some key exceptions:
  - When using the DLTA/CFD method, the steady solution for the shock location/magnitude can present indirect implications in the LCO prediction.
  - The heating of the plate is due to the aerodynamic flow: The thermal field needs to be determined by a RANS model or from experiment. Piston theory, potential flow theory and Euler flow all give similar results, but all require some auxiliary modeling of the heating due to the aerodynamic flow (e.g. Eckert model).

The computational models agree with the observations from experiments **on the essentials of the physical phenomena** e.g. buckling, flutter and limit cycle oscillations.

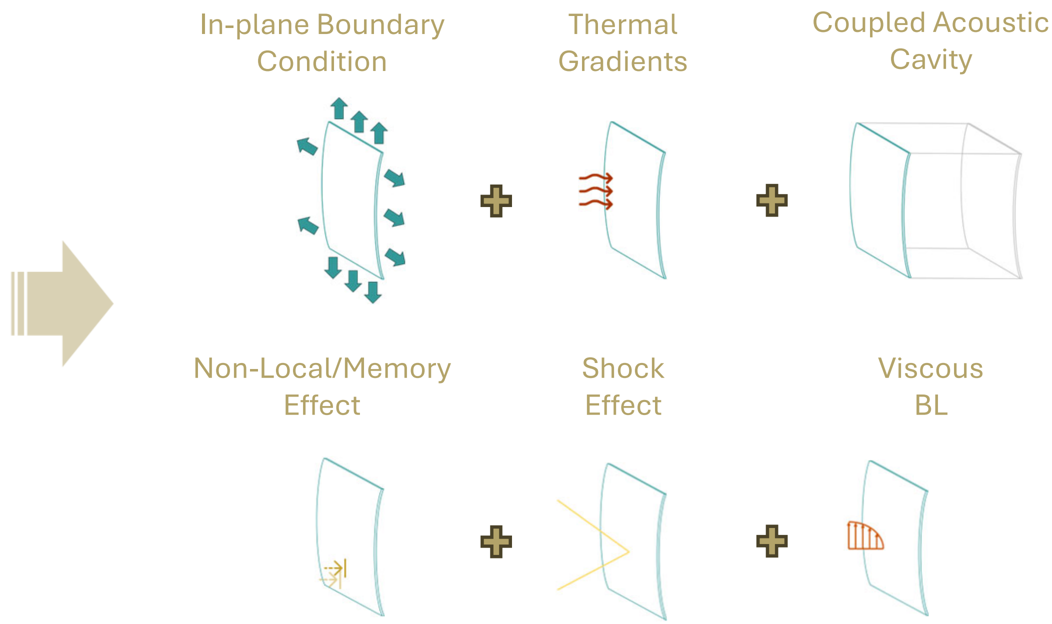
There is broad quantitative agreement between computations and experiments, given the sensitivity of the results to a wide range of parameters**.**

Motivation for this Work

- To provide the highest possible fidelity in the computational model at an affordable cost; orders of magnitude reduction in cost compared to traditional CFD/CSD methods
- To explore a wide range of relevant parameters including M_∞, Re, static pressure differential, thermal stresses and structural boundary conditions, both out of plane and in plane.
- **To correlate computational results with experimental results and assess the sensitivity of these results to uncertainties in key parameters**



Brouwer et al. AIAAJ (2021)



Mathematical/Computational Modeling

Nonlinear Aeroelastic Solver

Set Boundary Conditions

- p_C fixed
- ΔT fixed
- ICs
- Flow parameters

Compute Structural Matrices

- Geometrical dimensions
- β_{BC} varying per run

Compute Aerodynamic Matrices

- Piston Theory
- Potential Flow
- Numerical Methods (ROM): Dynamically Linearized Time-domain Approach

Solve ODE in terms of the generalized coord.

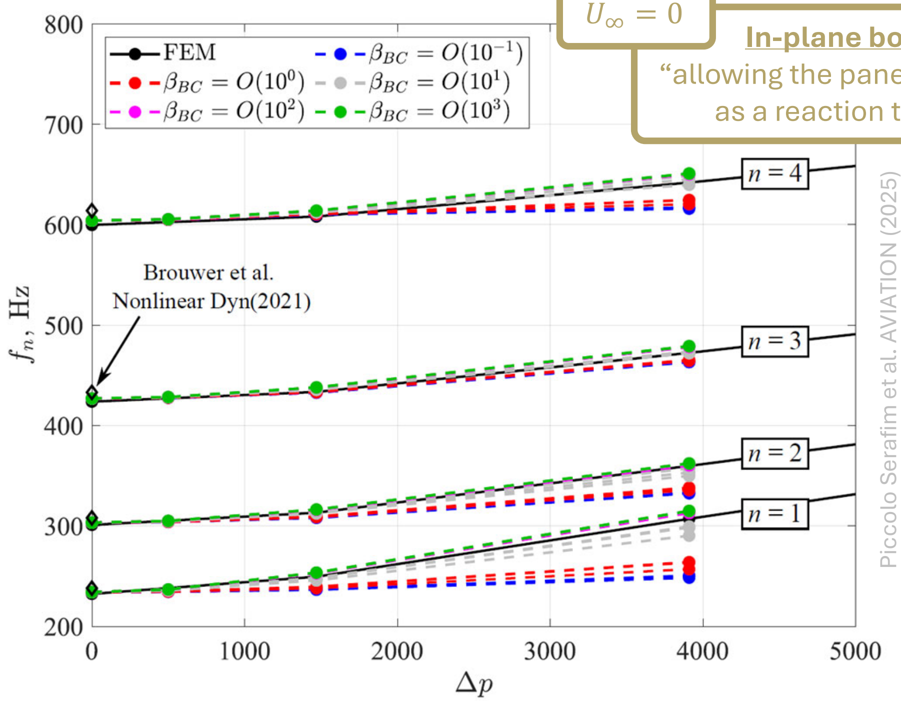
No acoustic influence

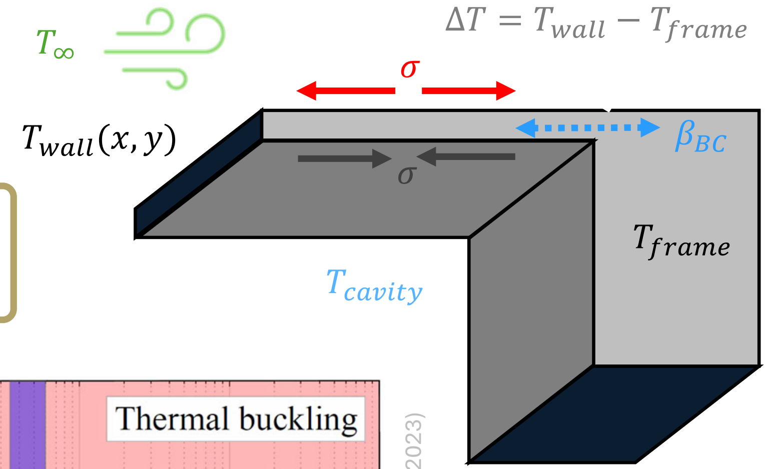
$$\underbrace{M_{mn}\ddot{q}_n(t) + C_{mn}\dot{q}_n(t) + G^{(2)}_{mn}q_n(t)}_{\text{Linear plate model}} + \underbrace{D^{(2)}_{mnrp}q_n(t)q_r(t)q_p(t)}_{\text{NL structural stiffness}} + \underbrace{Q_m^{Aero}}_{\text{Aero}} + \underbrace{Q_m^{static}}_{\text{Static pressure differential}} = 0$$

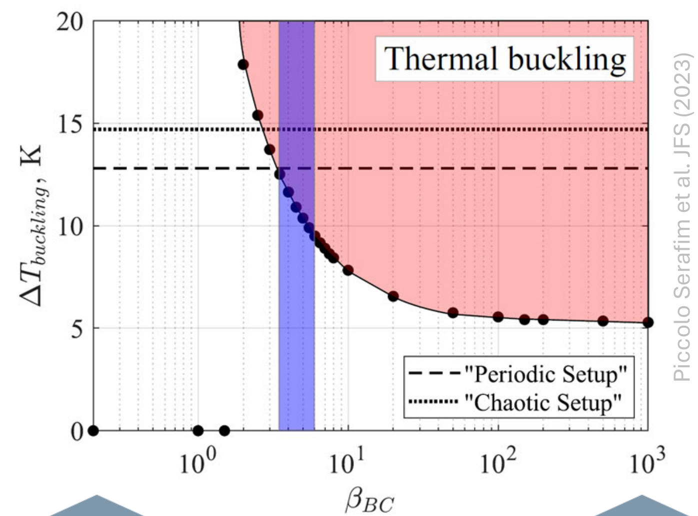
In-plane Boundary Condition & Temperature Gradient ΔT

$\Delta T = 0$
$U_\infty = 0$

In-plane boundary stiffness
"allowing the panel to deform at the edges as a reaction to the fixed support"







Expected range for the RC-19 conf. :
$O(10^1) < \beta_{BC} < O(10^2)$

Free edge displacement
↓
No buckled condition

No edge displacement
↓
Buckled condition

Additional Considerations: Heat Equation (HE) Implementation

$$\underbrace{M_{m,n}\ddot{q}_n(t) + C_{m,n}\dot{q}_n(t) + G^{(1)}_{m,n}q_n(t)}_{\text{Linear plate model}} + \underbrace{D^{(2)}_{m,n,r,p}q_n(t)q_r(t)q_p(t)}_{\text{NL structural stiffness}} + \underbrace{G^{(3)}_{m,n,r}q_n(t)T_r(t)}_{\text{Thermal coupling}} + \underbrace{Q^{Aero}_{m,n} + Q^{static}_{m}}_{\text{Pressure terms}} = 0$$

$$\underbrace{M^H_{m,n}\dot{T}_n(t) + K^H_{m,n}T_n(t)}_{\text{Thermal inertia and stiffness}} + \underbrace{Q^0_m + Q^q_{m,n}q_n(t) + Q^{\dot{q}}_{m,n}\dot{q}_n(t) + Q^{Tq}_{m,n}T_n(t)}_{\text{Coupled linear aerodynamic heating}} = 0$$

The ΔT measured by the thermocouple is slightly smaller than the mid-plate temperature

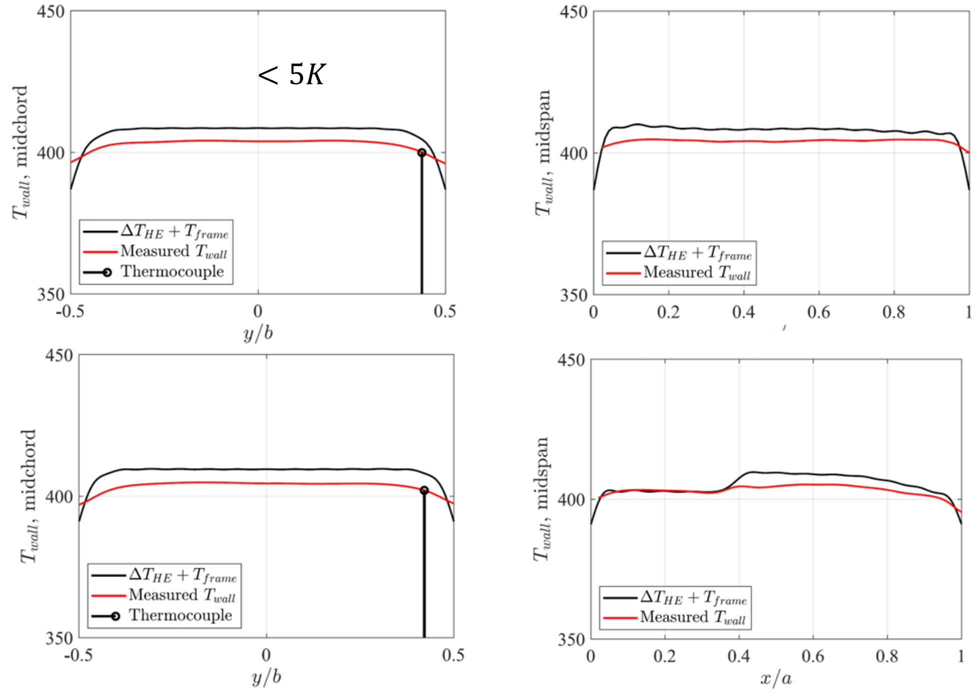
The HE implementation overpredicts the mid-plate temperature by $< 5K$

Wind Tunnel Setup

Parameter	Value
M_∞	1.92
p_0	≈ 346 kPa
T_0	≈ 420 K
Δp	0

No shock

4° Wedge Shock



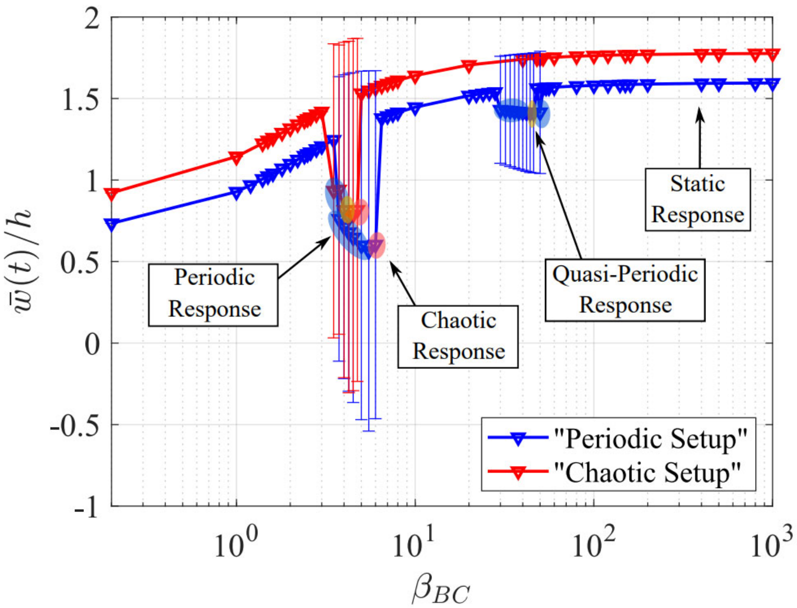
Different setup from the workshop case!

No-shock: Periodic and Chaotic LCOs

Periodic Parameters		Chaotic Parameters	
Δp (kPa)	3.91	Δp (kPa)	5.01
ΔT (K)	12.8	ΔT (K)	14.7

β_{BC} can be determined from a ground vibration test of Finite Element Modeling of panel + its boundary support

$$\beta_{BC} \equiv \frac{K_{BC}a}{Eh}$$

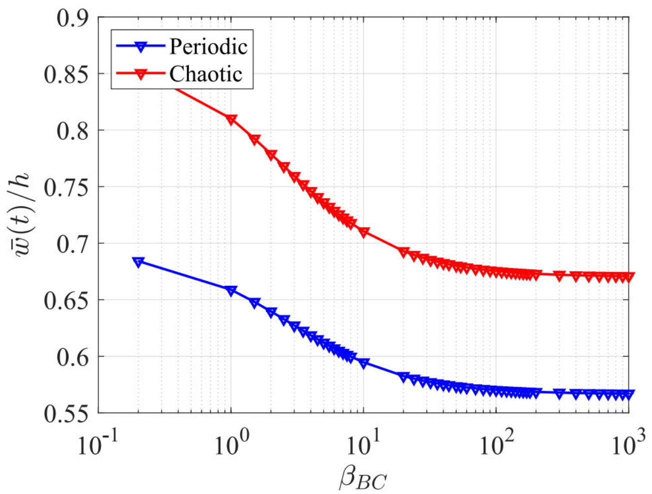


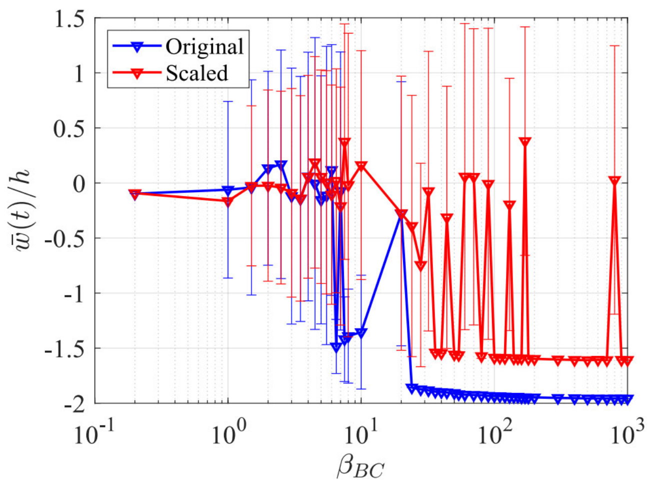
Piccolo Serafim et al. JFS(2023)

Model Sensitivity

	$\Delta p = 0$	Nominal Δp (Periodic/Chaotic)	Δp from NASA CFD Solution Profile	Δp from Exp. Data
$\Delta T = 0$	-	Steady Deformation	Steady Deformation	LCO (All range of β_{BC})
Nominal ΔT (Periodic/ Chaotic)	LCO (All range of β_{BC})	LCO (Limited range of β_{BC})	LCO (Limited range of β_{BC})	?
ΔT from Heat Equation	LCO (All range of β_{BC})	Steady Deformation	LCO (All range of β_{BC})	LCO (All range of β_{BC})

No-shock: Periodic and Chaotic LCOs





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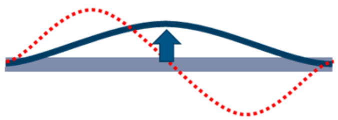
ROM to include unsteady aerodynamics into the aeroelastic solution

Step change in the CFD domain for each ψ_{mn} …

… in the panel deformation:



1^{st} set of CFD runs

… in the velocity of the panel deformation:

2^{nd} set of CFD runs

… to obtain a pressure field (output) from a known deformation (input)

$$Q_{mn}^{CFD}(t) = \iint \frac{\Delta p_m(x,y,t)}{\rho_\infty U_\infty^2} \psi_n(x,y)\,dxdy$$

Work the other way around to obtain the A_{mn}, B_{mn}, and $E_{mn}(t)$ using the input/output relationship

$$Q_{mn}^{CFD}(t) = q_m(t)A_{mn} + \dot{q}_m(t)B_{mn} + \int_0^t q_m(\tau)E_{mn}(t-\tau)d\tau$$

Once A_{mn}, B_{mn}, and $E_{mn}(t)$ are obtained, we can reconstruct the Generalized Aerodynamic Force inside the Aeroelastic Solver for any arbitrary panel deformation ($q_m(t)$, $\dot{q}_m(t)$)

$$Q_{mn}(t) = q_m(t)A_{mn} + \dot{q}_m(t)B_{mn} + \int_0^t q_m(\tau)E_{mn}(t-\tau)d\tau$$

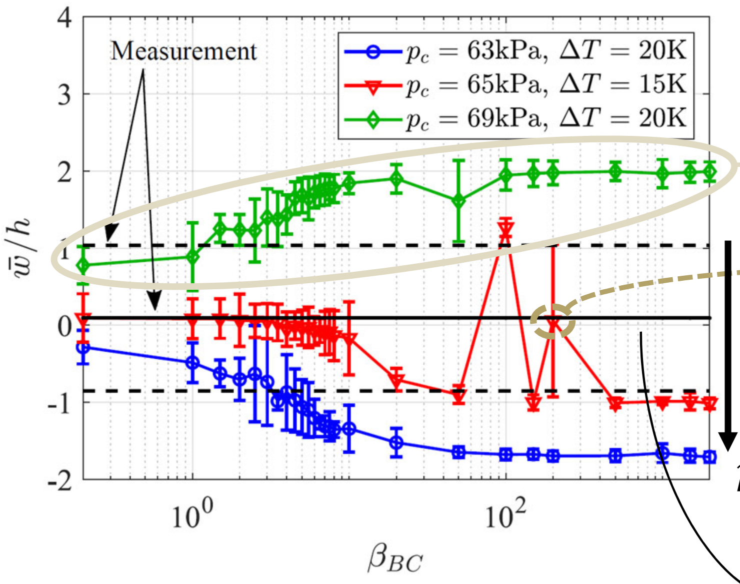
DLTA with an Inviscid Shock Impingement

$\theta = 4^{\circ}$ wedge shock configuration

Wind Tunnel Setting	
p_c (kPa)	ΔT (K)
68.9	13.3

Mean & SDT deformation

(at ¾ of the panel length)



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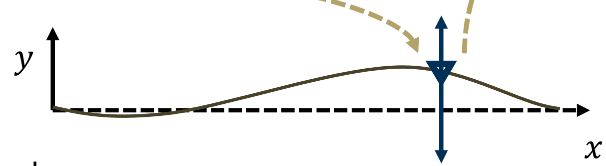
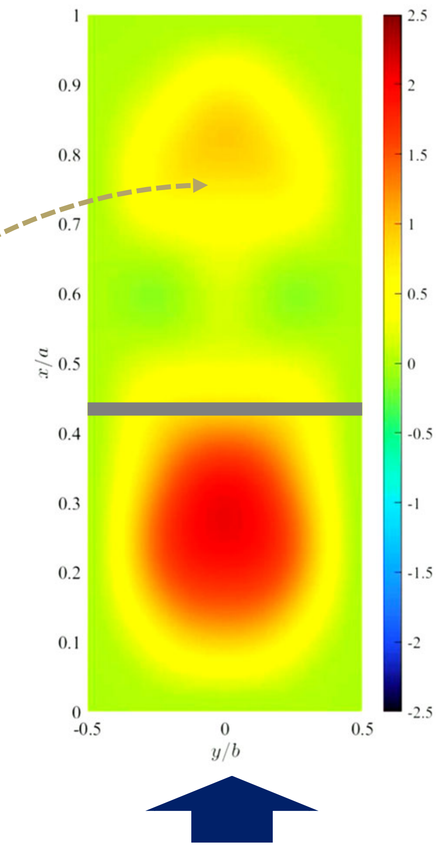
Similar settings as in the wind tunnel



Measurement Avg.



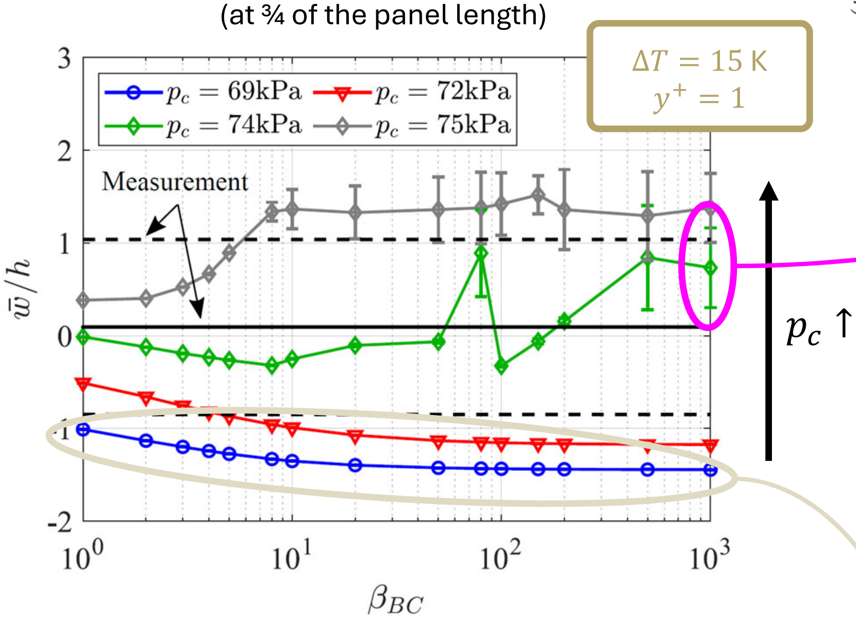
Flow Field

DLTA with a Viscous Shock Impingement

$\theta = 4^{\circ}$ wedge shock configuration

Mean & SDT deformation

(at ¾ of the panel length)



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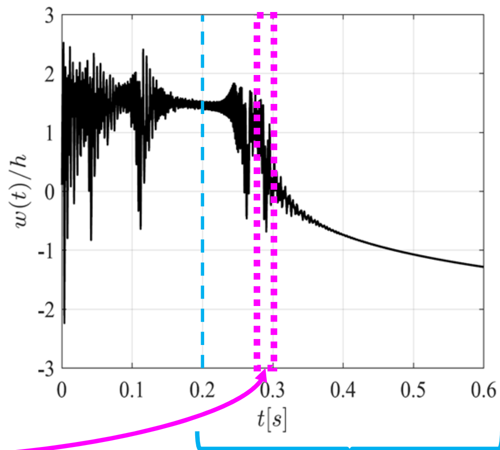

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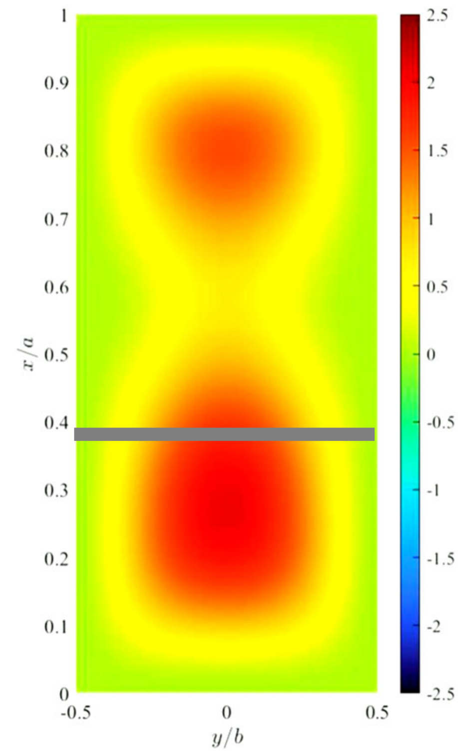
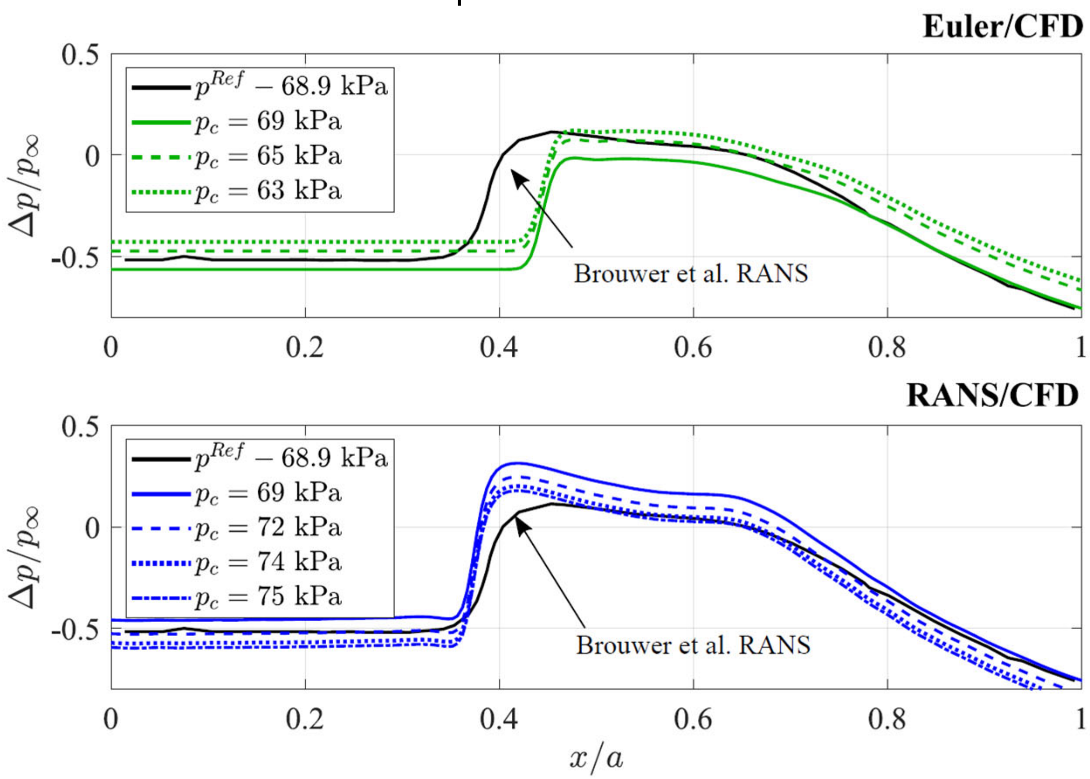


Flow Field

Shock Impingement Aerodynamic Properties

Steady Solution

Static pressure differential



Panel stiffened by the Δp effect

Approx. Run time for each Aero. Model

For one set of parameters, for each value of in-plane BC ($\boldsymbol{\beta}_{BC}$)

Different modes running in parallel

Aerodynamic Case	Analytical Solution	CFD	CFD Run time	LCO run time per $\boldsymbol{\beta}_{BC}$		Pros	Cons
				Local Computer	Cluster		
Piston Theory	X	-	-	< day	A couple of hours	Really fast	Local No shock case Inviscid
Potential Flow	X	-	-	~1 day*	< day*	Non-local Relative fast	No shock case Inviscid
Euler/CFD	-	X	~2-3 days	-	< day**	Shock case Potential for RANS	Takes more time to run
RANS/CFD	-	X	~3-4 days	-	< day**	Shock case Viscosity effects	Takes more time to run

* if the influence matrices are not previously saved
** if the pre-CFD matrices are previously performed

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