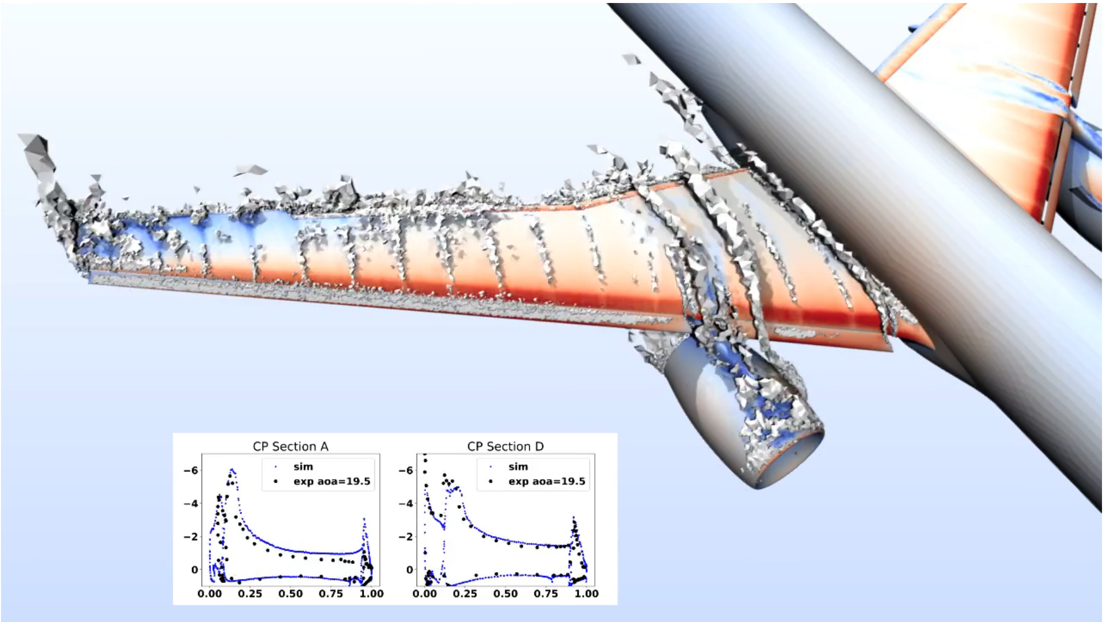
Adaptive Euler presentation for AePW-DPW Workshop 2026 by Johan Jansson (jjan@kth.se). The presentation is a HTML presentation (but might be exported in PDF form) and includes animations which likely only work in HTML form, please view the presentation with this link: https://digitalmath.tech/digitalmath/aepw-dpw_workshop2026_AdaptiveEuler.html

Collaborative effort together with SAAB (Pär Gustafsson, Anders Karlsson) and FOI (Olivier Amoignon, Kristoffer Wingstedt, Christian Helanow)

Meshes from Helden (Andrew Wick)



Adaptive Euler - resolution of the Grand Challenge of aerodynamics: first principles, automated computation and error control in FEniCS, validation in HLPW, and prototype development toward aeroelasticity and FSI

- Today the industry does not trust aerodynamics simulation - calls it "Colorful Fluid Dynamics"
- Euler aerodynamics makes simulation **reliable** - direct simulation of natural laws
- You can now reliably design, optimize, certify and innovate aircraft, wind turbines, ships, etc. in the web browser, **100x faster and cheaper than industry standard.**
- **Strong validation and recognition** of breakthrough Euler aerodynamics at the highest level benchmarking workshops (NASA HLPW), Formula 1, etc.
- Reproducible Open Source Digital Math/FEniCS setting is **future-proof** - editable, testable, extensible - see e.g. how Linux (with Red Hat as key commercial actor) has become fundamental technology that large part of industry is built on.
- Delivered as DigiMat online course, largest online course in Sweden with 30000+ participants.

Johan Jansson (**jjan@kth.se**), key collaboration with Kristoffer Wingstedt (prev. Saab AB now FOI)
http://icarusmath.com
http://digitalmath.org (research environment)

*Leonhard Euler [1707-1783] one of the greatest mathematicians in history.

Methodology

- **Proposed Adaptive Euler as solution to the NASA Vision 2030 challenges.**
- **Predictive incompressible and compressible Euler (two separate formulations) - first principles - no modeling parameters - free slip BC.**
- **Direct FEM cG(1)cG(1) with GLS stabilization in FEniCS with adjoint-based adaptive error control.**
- Extremely fast and cheap: < 100 core hours - "zero" computational cost. 300x cheaper than industry standard "fastest" simulation technology RANS, 1000x-10000x cheaper than WMLES ("scale-resolving" technology) [incompressible - we will generate compressible results in AePW-4/DPW-8]
- Coarse starting meshes for adaptive methodology from ANSA/Helden: 100k vertices giving good validation.
- Vision: Design and certification of aircraft and engineering systems should be done by simulation (e.g. Digital Math Real Flight Simulator) within 5 years.

Adaptive Euler DPW/AePW highlights and plan 1/3

- We emphasize our results are still a "development snapshot", we are developing and testing a new formulation.
- Ongoing uncertainty quantification.
- Case 1: We capture the main "transsonic buffet" phenomenon (large-magnitude shock oscillation) with Euler - free slip, zero friction, no boundary layer.
  Similar oscillation magnitude of appx. 5-10% of chord as in the main Jaquin et. al. reference.
  See e.g. the main Jaquin et. al. reference: "In transonic flow conditions, shock wave/turbulent boundary-layer interaction with separation may induce large- scale instabilities involving the whole flowfield and is known as transonic buffet"
  Our working hypothesis based on our very prelimniary results is that "transonic buffet" is not connected to the boundary layer, since we have no boundary layer.
- We appear to keep the very high efficiency we demonstrated in HLPW5, extremely coarse meshes and very large time steps, the total computational cost is appx. 1000 core hours for one simulation.
- **Case 2 status: Very preliminary promising results with same high efficiency as shown in HLPW5 and for AePW/DPW Case 1. Extremely coarse mesh with appx. 100k vertices. We appear to capture the key stationary qualitative (very preliminary) and (very preliminary) quantitative phenomena, and we present a very preliminary simulation where we capture what could be a buffet phenomenon. The total computational cost is appx. 1000 core hours for one simulation.**
- In HLPW5 we demonstrated that we capture "wing wake-tail buffet", which is also of key importance for e.g. delta wings such as the FA-18 Hornet.

Adaptive Euler DPW/AePW highlights and plan 2/3

- Case 3 plan:
  Bottom-Up Simple FSI: Iterate between Compressible Adaptive Euler fluid formulation and a simple volume linear elastic formulation (also in FEniCS), transferring pressure and deformation, with elastic mesh smoothing. **Very preliminary promising two-way FSI results for Case 3** with same high efficiency as shown in HLPW5 and for AePW/DPW Case 1-2. Extremely coarse mesh with appx. 100k vertices. We appear to capture the key qualitative phenomena: shock/CP similar to Case 2, and the expected qualitative wing oscillation/deformion.

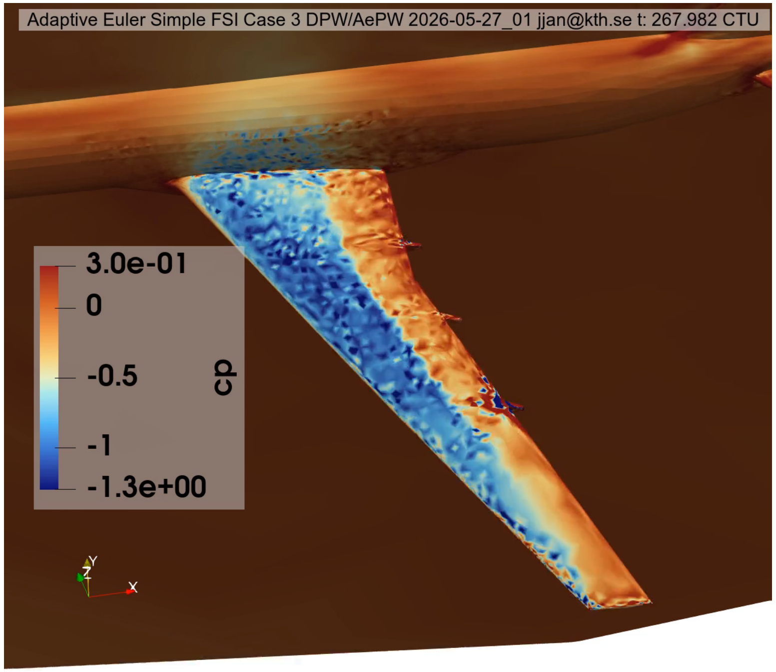
  Top-Down: Use our previously developed Unified Continuum FSI (UC-FSI) methodology/formulation with Adaptive Euler in the fluid phase. UC-FSI is simply a generalization of Adaptive Euler where we introduce a phase marker which selects constitutive law, and convection of the phase marker. Otherwise the methodology/formulation is the same: GLS-stabilized P1 space-time FEM in FEniCS with adjoint-based adaptive error control. UC-FSI means we discretize and solve for the entire momentum, density and energy fields for the entire FSI domain as one system - stabilization and adaptive error control automated and same for the entire system. Challenge to represent the discontinuity in the FSI interface in Adaptive Euler. We have a large body of previous work for UC-FSI for biomechanics where the FSI interface is continuous.

  Preliminary working Immersed Adaptive Euler suitable for Unified Continuum FSI.
  Collaboration with Susanne Claus (ONERA) who is a main developer of CutFEM in FEniCS (can be seen as more accurate quadrature/representations for immersed methods), and generation of immersed geometry representations.

Adaptive Euler DPW/AePW highlights and plan 3/3

- Johan Jansson presented an overview of our Adaptive Euler activity in HLPW/DPW/AePW and industry activity at the FT2025 conference:
  https://ft2025.se/
  https://beyond-production.objects.dc-sto1.glesys.net/projects/9050e852-2137-4dd2-af9d-83d0262fb81f/documents/No8hATgof3fLaW8FdhrRVVnngfSgChDiogon3BJc.html

Adaptive Euler Compressible Simple Two-way FSI AePW-4 Test Case 3 CRM very preliminary validation Variant 2026-05-31 01



Observations: Similar shock structure and CP (very preliminary) as Case 2 (fixed geometry), and the expected qualitative wing oscillation/deformation. We observe the characteristic buffet phenomenon of pressure oscillation.
We use starting meshes from Helden (see following slides).

We expect more mature results in follow-up activities to the workshop, the results here can be seen as a showcase of the framework as a test bed for FSI/Case 3.

Direct FEM simulation of the compressible Euler equations with GLS stabilization in FEniCS - first principles - free slip - no modeling parameters..

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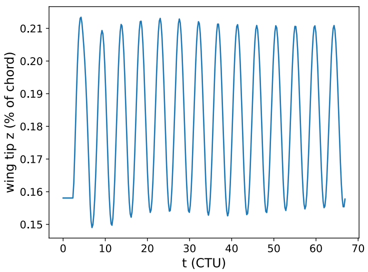

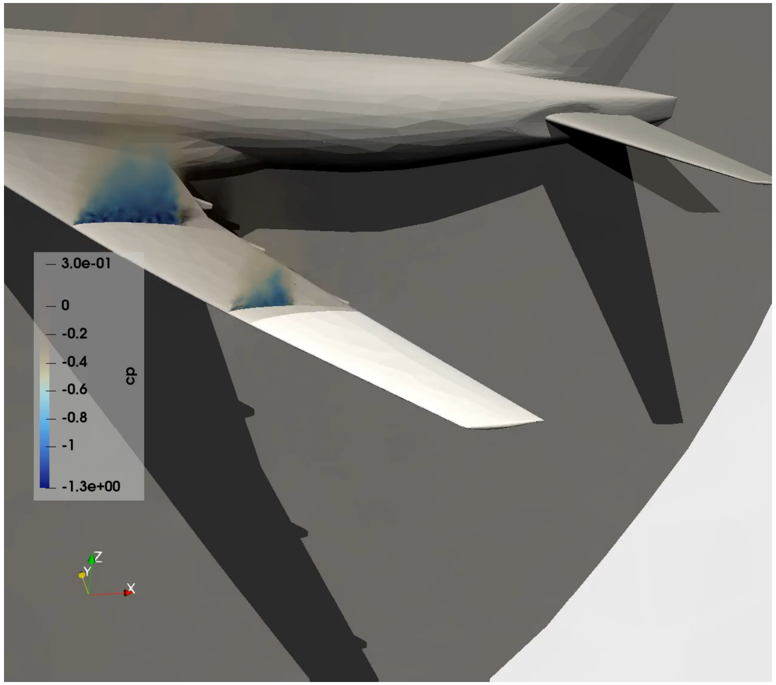

(Slice through Eta=0.5 and Eta=0.8 of full CRM aircraft (from previous slide))
Observations: Similar shock structure and CP (very preliminary) as Case 2 (fixed geometry), and the expected qualitative wing oscillation/deformation. We observe the characteristic buffet phenomenon of pressure oscillation.
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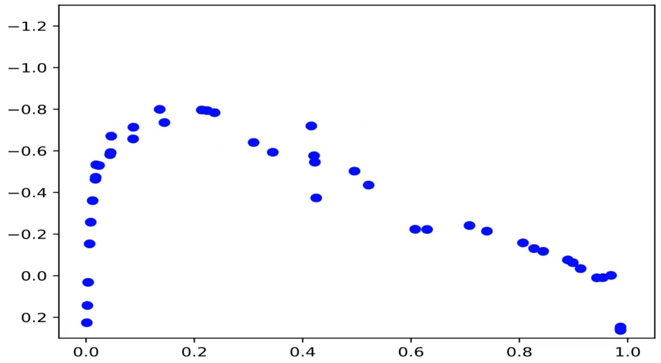

(Animated CP cut through Eta=0.8 of full CRM aircraft (from previous slide))
Observations: Similar shock structure and CP (very preliminary) as Case 2 (fixed geometry), and the expected qualitative wing oscillation/deformation. We observe the characteristic buffet phenomenon of pressure oscillation.
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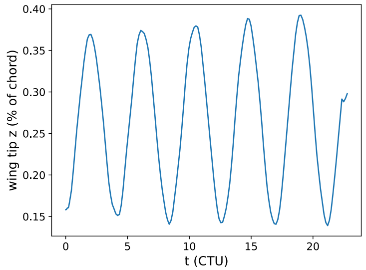

Very preliminary flutter verification. Increasing the density in a region outside Eta=0.8 causes the wing tip oscillation amplitude to increase in time without bound - consistent with flutter.
We use starting meshes from Helden (see following slides).

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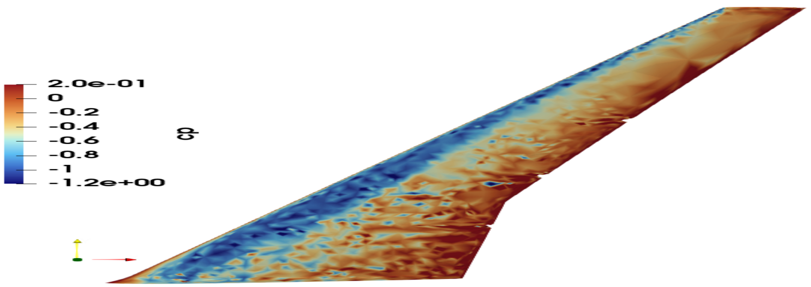
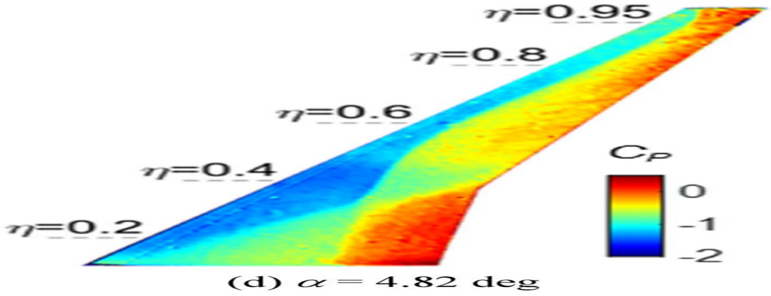
Direct FEM simulation of the compressible Euler equations with GLS stabilization in FEniCS - first principles - free slip - no modeling parameters..

Adaptive Euler Compressible AePW-4 Test Case 2 CRM very preliminary validation Variant 2025-07-29 01

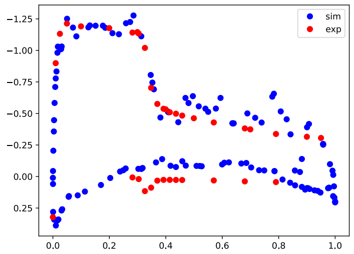




Similar shock structure and CP (very preliminary) (left), comparing to [Sugioka et. al. 2021] (right).
We use starting meshes from Helden, both for the Case 2 geometry and also from Case 2.1 from HLPW5 (CRM also with slat tracks and vertical stabilizer, which we modify ourselves, very similar to Case 2 geometry). We use meshes which have highest simulation efficiency, likely not significant difference, we will use workshop Case 2 geometry for final submission.

Direct FEM simulation of the compressible Euler equations with GLS stabilization in FEniCS - first principles - free slip - no modeling parameters..
Observations:
We appear to capture the key stationary qualitative (very preliminary) and (very preliminary) quantitative phenomena, we have not yet started investigating the buffet phenomena.

Adaptive Euler Compressible AePW-4 Test Case 2 CRM very preliminary validation Variant 2025-07-29 01



Similar shock structure and CP (very prelim).

Direct FEM simulation of the compressible Euler equations with GLS stabilization in FEniCS - first principles - free slip - no modeling parameters..
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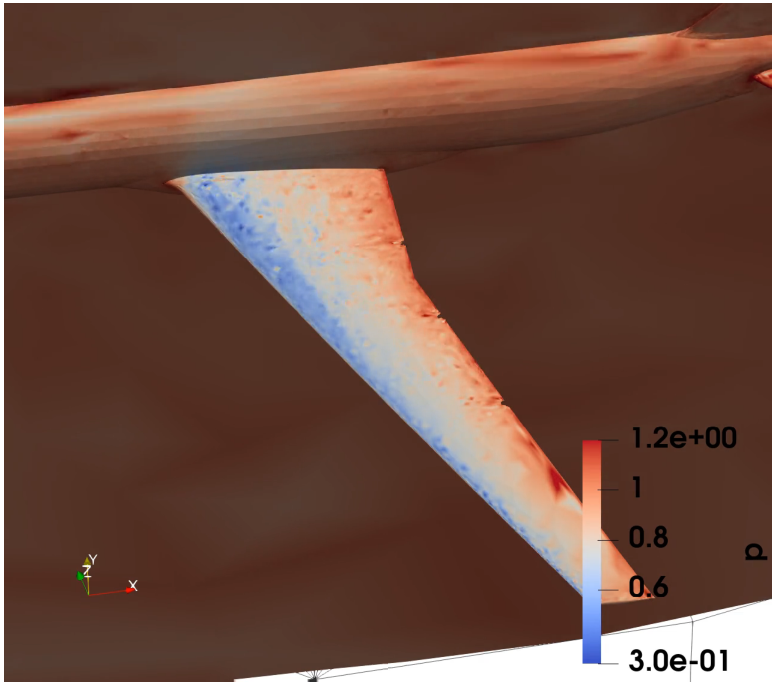
Adaptive Euler Compressible AePW-4 Test Case 2 CRM very preliminary validation Variant 2025-10-18 01



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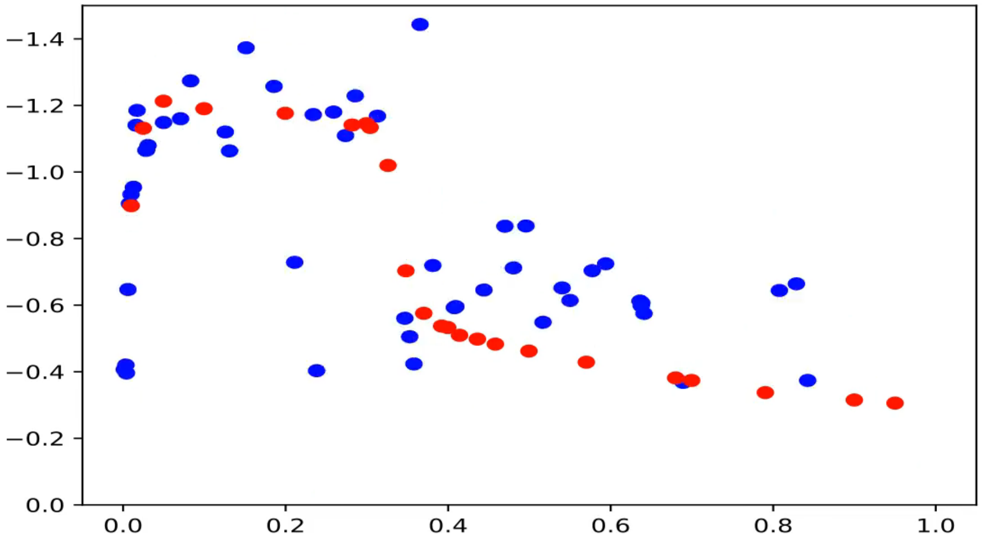

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You're welcome to get involved!
Try yourself, modify, extend!
http://digitalmath.tech/aero-breakthrough

Commercial cooperation:
http://icarusmath.com

For more details, see my presentations from our Adaptive Euler team as well as notes and other presentations from AePW-DPW WMLES&Beyond subgroup:
https://docs.google.com/document/d/13vwP_GxhCKznDaQR97zZxWmDcLFDNySdP-Cojxv4hBE/edit?usp=sharing

Any questions, ideas, comments:
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