Comparison of SA Turbulence Models using the ONERA OAT15A Airfoil

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Numerical Overview

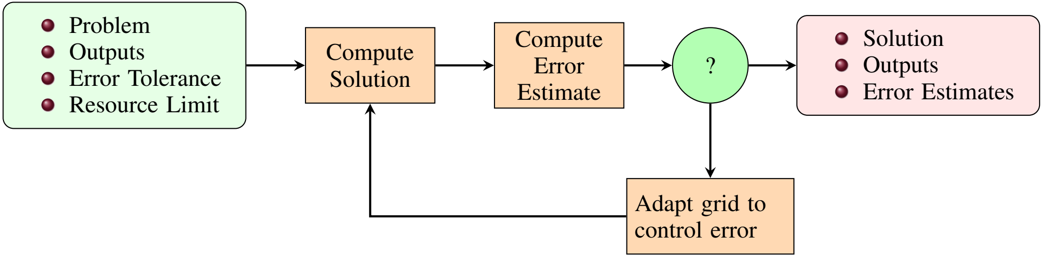
Solver:

- Solution Adaptive Numerical Simulator (SANS)
- Adjoint Consistent Finite Element Method
  - Variational Multi-Scales with Discontinuous subscales (VMSD)
- 2^{nd}-order accuracy (high-order supported)
- Quasi-Newton Pseudo-Time Continuation
  - CFL: $1 \rightarrow \infty \; (10^9) \Rightarrow$ Newton

Focus here:

- Output-based Mesh Adaptation
  - Eliminate numerical error for wall model comparison
- Compare at SA-neg vs. SA-neg-QCR2000 for ONERA OAT15A airfoil

Mesh Refinement via an Adaptive Process



- Error estimates via Dual Weighted Residual (Adjoint) method
- MOESS Mesh Optimization (Yano, PhD Dissertation, MIT 2012):
  - Find mesh that minimizes output error for a target Degree of Freedom (DOF) count
- Agnostic to discretization order of accuracy and PDE
- Provably optimal meshes (Carson, PhD Dissertation, MIT 2020)
- Implemented with Solution Adaptive Numerical Simulator (SANS)

SA-neg vs. SA-neg-QCR2000

- QCR2000 modifies Boussinesq assumption of Reynolds stresses, $\bar{\bar{\tau}}_t$
- Modified mean flow equations with $\bar{\bar{O}}$ antisymmetric normalized rotation tensor
- Turbulence model PDE unchanged

SA-neg

$$\text{Cont.}\quad \frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{q}) = 0$$

$$\text{Mom.}\quad \frac{\partial \rho\vec{q}}{\partial t} + \nabla\cdot\left(\rho\vec{q}\,\vec{q}\right) = -\nabla p + \nabla\cdot\left[\bar{\bar{\tau}} + \bar{\bar{\tau}}_t\right]$$

$$\text{Engy.}\quad \frac{\partial}{\partial t}(\rho E) + \nabla\cdot\left(\rho\vec{q}H\right) = \nabla\cdot\left(k\nabla T + \vec{q}\cdot\left[\bar{\bar{\tau}} + \bar{\bar{\tau}}_t\right]\right)$$

$$\text{SA.}\quad \frac{\partial \rho\tilde{\nu}}{\partial t} + \nabla\cdot\left(\rho\vec{q}\tilde{\nu}\right) = \rho[P - D] + \frac{1}{\sigma}\left[\nabla\cdot\left[\rho(\nu + \tilde{\nu}f_n)\nabla\tilde{\nu}\right] + c_{b2}(\nabla\tilde{\nu})^2 - (\nu + \tilde{\nu}f_n)\nabla\rho\cdot\nabla\tilde{\nu}\right]$$

- QCR2000 modifies Boussinesq assumption of Reynolds stresses, $\bar{\bar{\tau}}_t$
- Modified mean flow equations with $\bar{\bar{O}}$ antisymmetric normalized rotation tensor
- Turbulence model PDE unchanged

SA-neg-QCR2000

$$\text{Cont.}\quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = 0$$

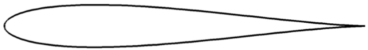
$$\text{Mom.}\quad \frac{\partial \rho \vec{q}}{\partial t} + \nabla \cdot \left(\rho \vec{q}\,\vec{q}\right) = -\nabla p + \nabla \cdot \left[\bar{\bar{\tau}} + \bar{\bar{\tau}}_t - C_{cr1}\left[\bar{\bar{O}}\bar{\bar{\tau}}_t + \left(\bar{\bar{O}}\bar{\bar{\tau}}_t\right)^t\right]\right]$$

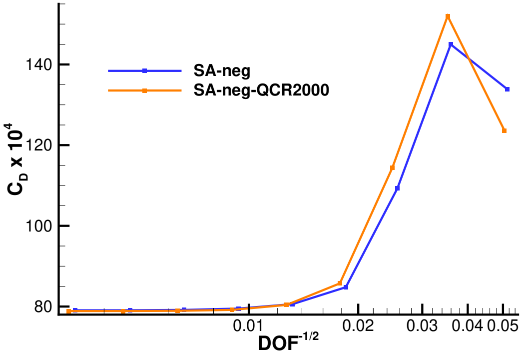
$$\text{Engy.}\quad \frac{\partial}{\partial t}(\rho E) + \nabla \cdot \left(\rho \vec{q} H\right) = \nabla \cdot \left(k \nabla T + \vec{q} \cdot \left[\bar{\bar{\tau}} + \bar{\bar{\tau}}_t - C_{cr1}\left[\bar{\bar{O}}\bar{\bar{\tau}}_t + \left(\bar{\bar{O}}\bar{\bar{\tau}}_t\right)^t\right]\right]\right)$$

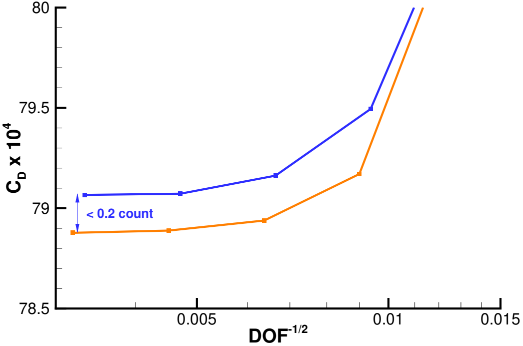
$$\text{SA.}\quad \frac{\partial \rho \tilde{\nu}}{\partial t} + \nabla \cdot \left(\rho \vec{q} \tilde{\nu}\right) = \rho[P - D] + \frac{1}{\sigma}\left[\nabla \cdot \left[\rho(\nu + \tilde{\nu} f_n)\nabla \tilde{\nu}\right] + c_{b2}(\nabla \tilde{\nu})^2 - (\nu + \tilde{\nu} f_n)\nabla \rho \cdot \nabla \tilde{\nu}\right]$$

Subsonic flow

- QCR2000 has small impact on drag



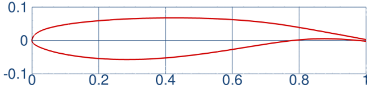




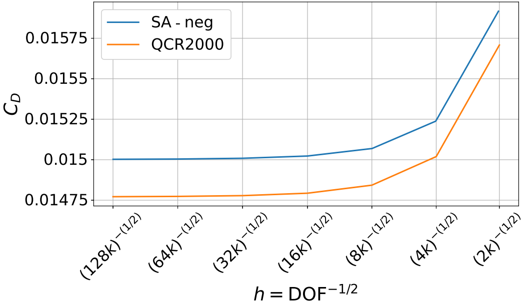
SA-neg vs. SA-neg-QCR2000 for OAT15A $\alpha = 1.5°$

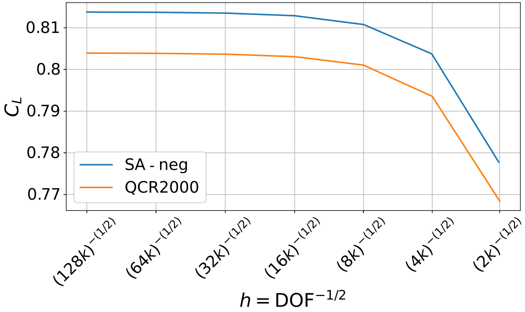
Transonic flow

- QCR2000 has impact on forces
- 2.5 counts



ONERA OAT15A Transonic Airfoil

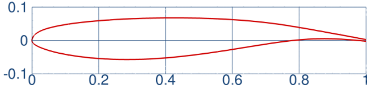




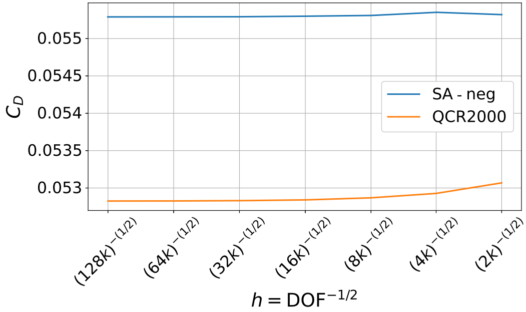
SA-neg vs. SA-neg-QCR2000 for OAT15A $\alpha = 3.9°$

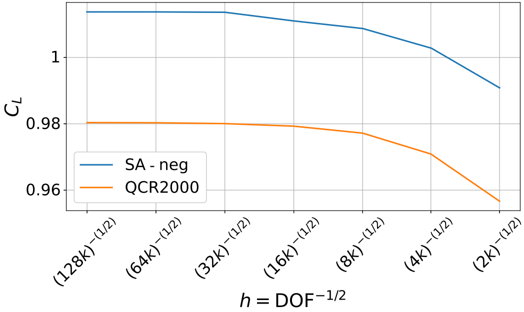
Transonic flow

- QCR2000 impact increases with α
- 25 counts

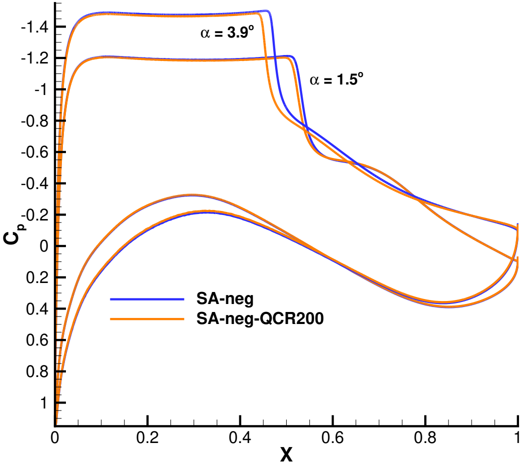


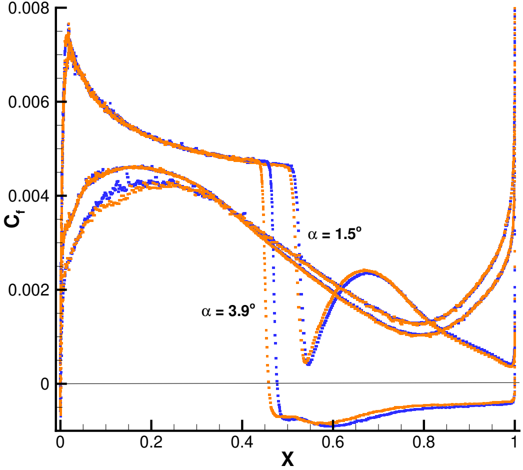
ONERA OAT15A Transonic Airfoil





SA-neg vs. SA-neg-QCR2000 for OAT15A C_p and C_f





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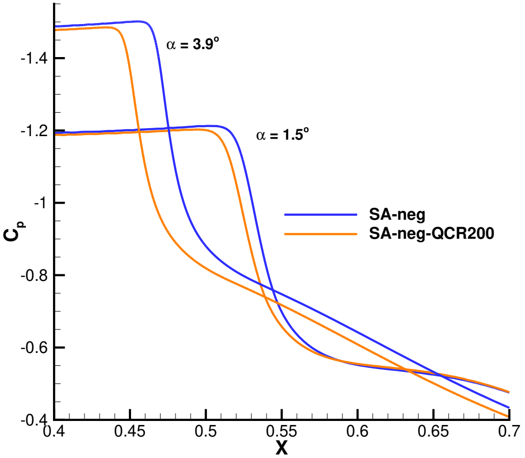
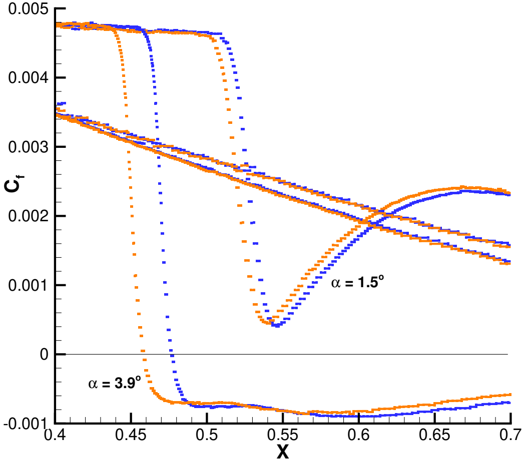
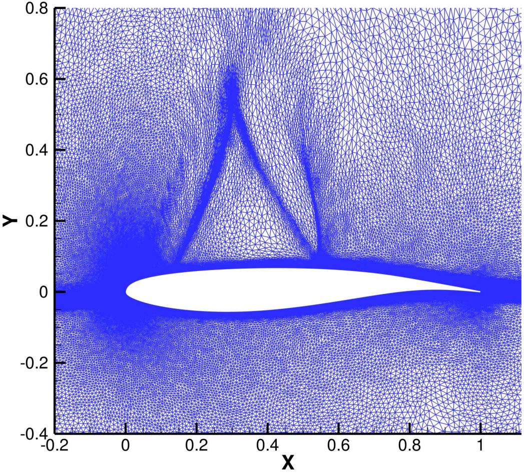


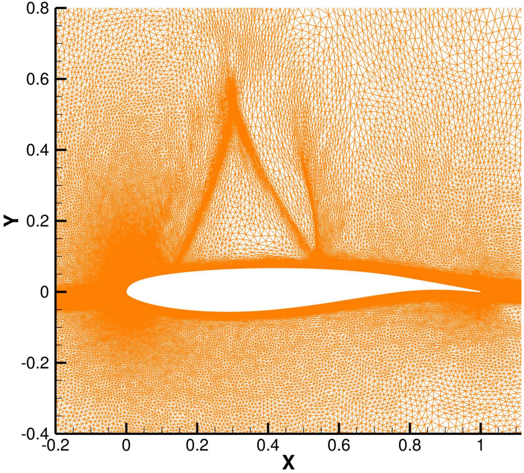


SA-neg vs. SA-neg-QCR2000 for OAT15A $\alpha = 1.5°$

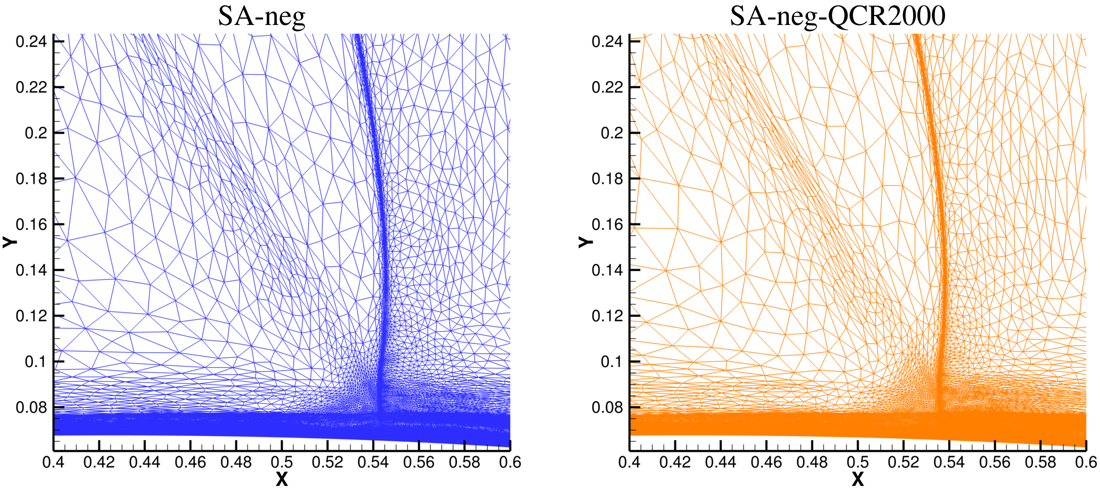
SA-neg



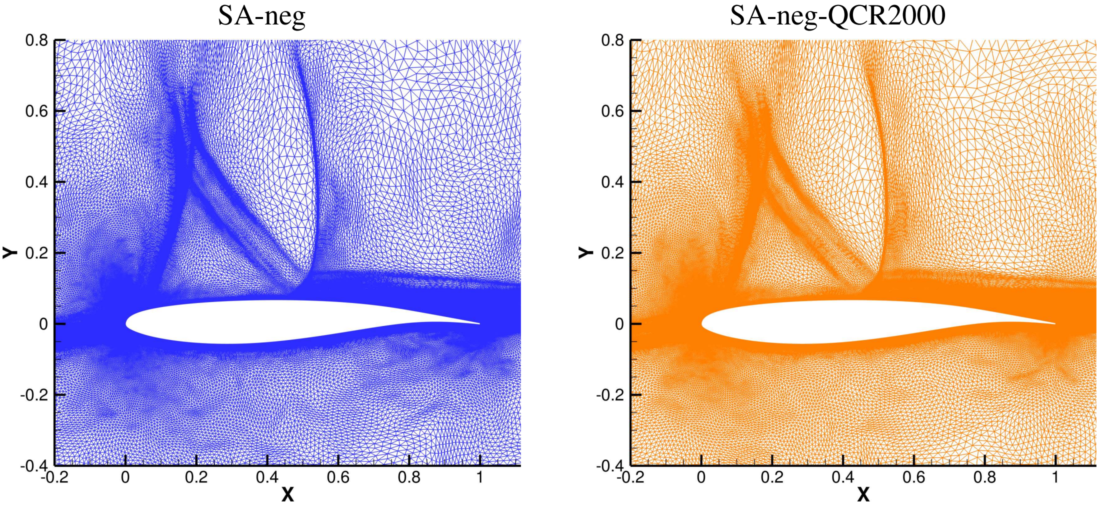
SA-neg-QCR2000



SA-neg vs. SA-neg-QCR2000 for OAT15A $\alpha = 1.5°$

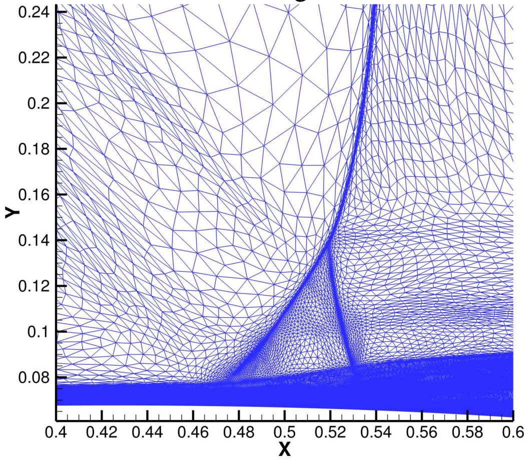


SA-neg vs. SA-neg-QCR2000 for OAT15A $\alpha = 3.9°$

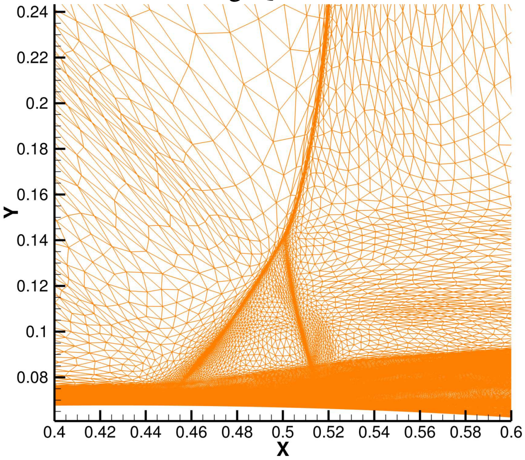


SA-neg vs. SA-neg-QCR2000 for OAT15A $\alpha = 3.9°$

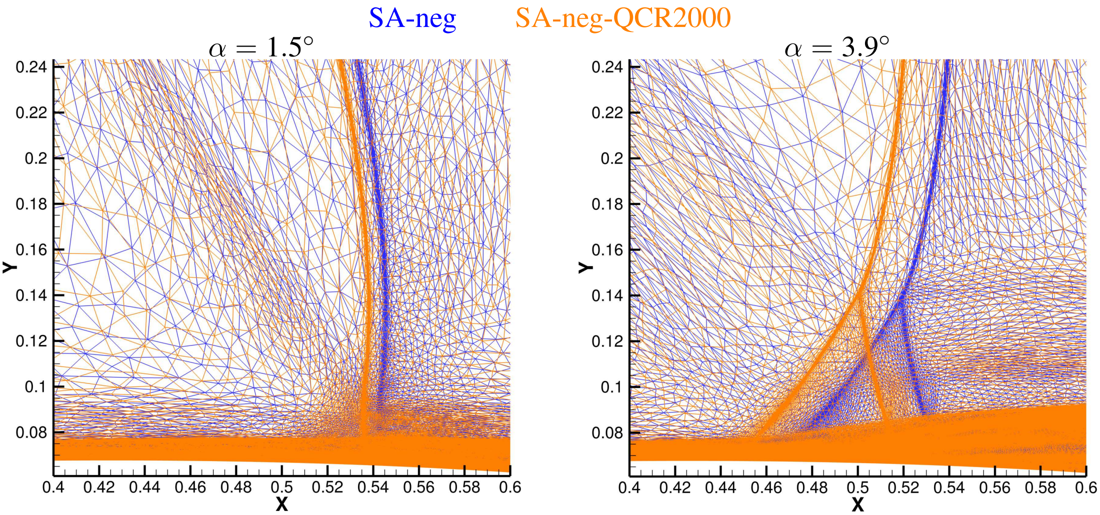
SA-neg



SA-neg-QCR2000



SA-neg vs. SA-neg-QCR2000 for OAT15A



Conclusion

- Output-based mesh adaptation to minimize (elimiate) numerical error
  - Enables inspection of modeling impacts
- Comparison of SA-neg vs. SA-neg-QCR2000
  - Little impact for subsonic airfoil
  - Larger impact for transonic airfoil
- These modeling differences contribute to ‘scatter’