On Flutter and the Limit Cycle Oscillation Envelope of the Pazy Wing using a Reduced Order Model

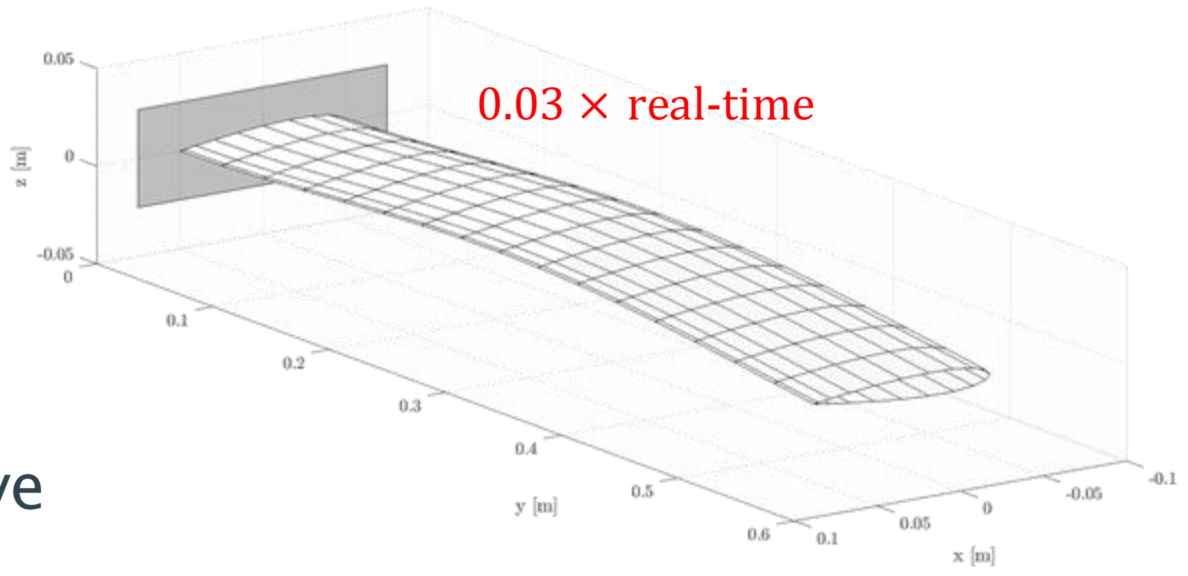
D. Clifford*, G. Morichetti, A. Da Ronch

23/07/2025

University of Southampton

Motivation

- Aeroelastic LCO prediction involves dynamic response analyses
  - Time integration of a large **nonlinear dynamical system**
  - Fluid nonlinearities & structural nonlinearities
- Process is computationally prohibitive for use in:
  - Parameter sweeps
  - Design search exploration
  - Uncertainty quantification



REDUCE COST WITH REDUCED ORDER MODELS (ROMs)

Objectives

- Reduced order models (ROMs) for rapid LCO prediction
  - **Dynamically nonlinear ROM**

- Demonstrate ROM capability with the **Pazy wing test-case**

Table of Contents

Projection Nonlinear Reduced Order Modelling

- **Project a Taylor expansion** of the FOM onto a subset of **aeroelastic eigenmodes**
  - **Data free**

- Simplified four step process

1. Determine reduced coordinate (subset of eigenmodes) ← **Linear ROM**
2. Taylor expansion of FOM in the **reduced coordinate**
3. Project Taylor expansion onto eigenmode subset

Retain dynamic nonlinearities

4. Time integrate the nonlinear ROM

Step 1: Reduced coordinate

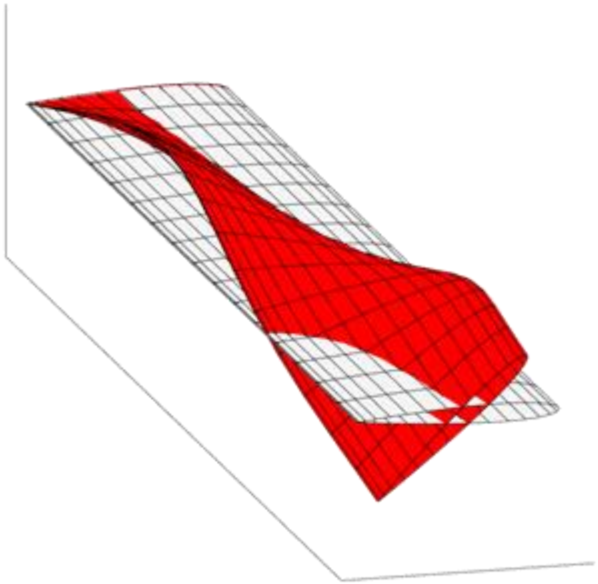
- Compute **left** and **right** eigenvalue problems of system Jacobian matrix
  - Select a subset, m, of the left and right eigenmodes

$$\boldsymbol{\Phi} = \begin{bmatrix} | & & | & & | \\ \boldsymbol{\phi}_1 & \cdots & \boldsymbol{\phi}_m & \cdots & \boldsymbol{\phi}_n \\ | & & | & & | \end{bmatrix} \quad \boldsymbol{\Psi} = \begin{bmatrix} | & & | & & | \\ \boldsymbol{\psi}_1 & \cdots & \boldsymbol{\psi}_m & \cdots & \boldsymbol{\psi}_n \\ | & & | & & | \end{bmatrix}$$

Right eigenmodes Left eigenmodes

- **ROM** coordinate transformation: $\boldsymbol{Z} \in \mathbb{C}^m$

$$\Delta \boldsymbol{X}(t) \approx \boldsymbol{\Phi}\mathbf{Z}(t) + \overline{\boldsymbol{\Phi}}\overline{\boldsymbol{Z}}(t)$$



Step 2: Taylor expansion in the reduced coordinate

- We do **NOT** want to compute **FOM** nonlinear terms!
  - Use **coordinate approximation** in Taylor expansion, $\boldsymbol{f}$

$$\boldsymbol{f}(\Delta\boldsymbol{X}) \rightarrow \boldsymbol{f}^{Z}(\boldsymbol{\Phi Z} + \overline{\boldsymbol{\Phi}}\overline{\boldsymbol{Z}})$$

$$\boxed{\frac{\partial^2 \boldsymbol{f}^Z}{\partial \boldsymbol{Z} \partial \boldsymbol{Z}} \in \mathbb{C}^{n \times m^2}}$$

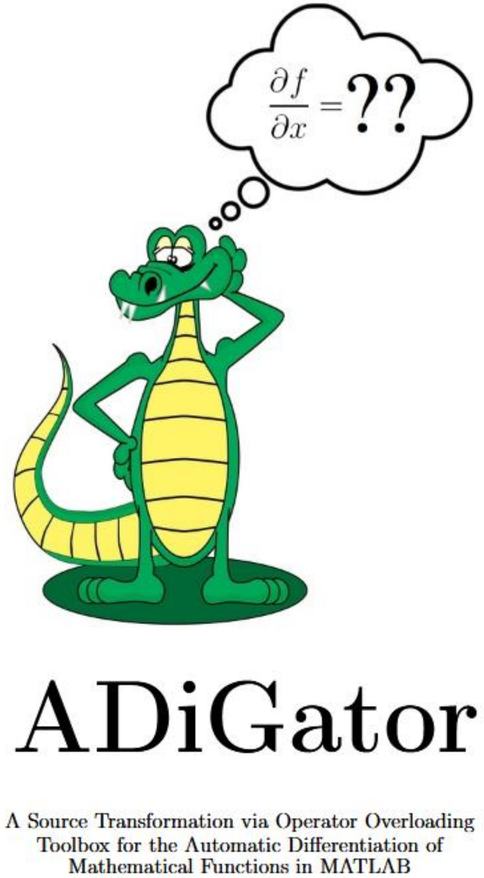
ROM Hessian

WE WANT THIS

$$\boxed{\frac{\partial^2 \boldsymbol{f}}{\partial \boldsymbol{X} \partial \boldsymbol{X}} \in \mathbb{R}^{n^3}}$$

FOM Hessian

WE **DO NOT** WANT THIS



- ROM derivatives computed using ADiGator: Source transformation AD

Step 3: Projection and hyper-reduction

- Project **ROM** nonlinear terms onto the reduced basis
  - Four quadratic **ROM** terms

$$\overline{\boldsymbol{\Psi}}^T \frac{\partial^2 \boldsymbol{f}^Z}{\partial \boldsymbol{Z} \partial \boldsymbol{Z}} \in \mathbb{C}^{m^3} \qquad \overline{\boldsymbol{\Psi}}^T \frac{\partial^2 \boldsymbol{f}^Z}{\partial \boldsymbol{Z} \partial \overline{\boldsymbol{Z}}} \in \mathbb{C}^{m^3} \qquad \overline{\boldsymbol{\Psi}}^T \frac{\partial^2 \boldsymbol{f}^Z}{\partial \overline{\boldsymbol{Z}} \partial \boldsymbol{Z}} \in \mathbb{C}^{m^3} \qquad \overline{\boldsymbol{\Psi}}^T \frac{\partial^2 \boldsymbol{f}^Z}{\partial \overline{\boldsymbol{Z}} \partial \overline{\boldsymbol{Z}}} \in \mathbb{C}^{m^3}$$

- Exploit symmetry of ROM nonlinear terms (hyper-reduction)

$$\boldsymbol{B}_{ROM} = \underbrace{\begin{bmatrix} \boxed{} \end{bmatrix}}_{4m^2} \Big\} m \longrightarrow \boldsymbol{B}_{ROM}^{sym} = \underbrace{\begin{bmatrix} \boxed{} \end{bmatrix}}_{2m^2 + m} \Big\} m$$

Step 4: Time integrate the nonlinear ROM

- Nonlinear **ROM** system of equations

$$\dot{Z} = \boxed{\Lambda Z} + \boxed{A_2^Z(Z, Z)} + \boxed{A_3^Z(Z, Z, Z)} + \cdots$$

ROM linear dynamics **ROM** nonlinear dynamics

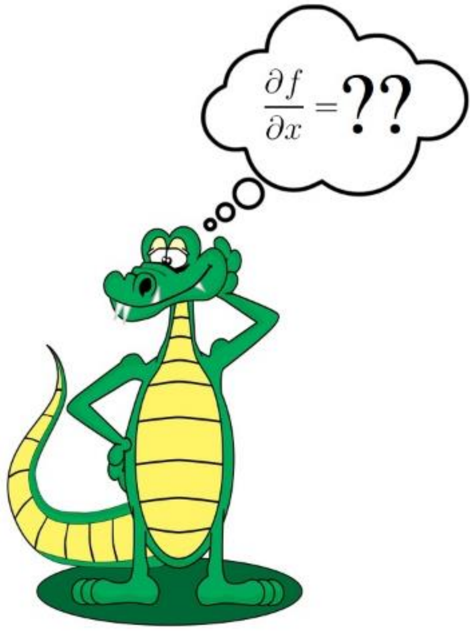
- Careful selection of the **ROM** modal bases can permit use of an explicit time integration scheme
  - RK4

Source-transformation automatic differentiation

- ROM **parameterised** via source transformation AD
  - One-off **OFFLINE** generation of codes for ROM quadratic/cubic nonlinear terms
  - **ONLINE** evaluation of codes at a fraction of the cost of generation

- **Source-transformation** AD efficient for large numbers of function evaluations: parameter sweeps

- Original projection nonlinear ROM used finite differencing, Woodgate (2007), Da Ronch (2012)



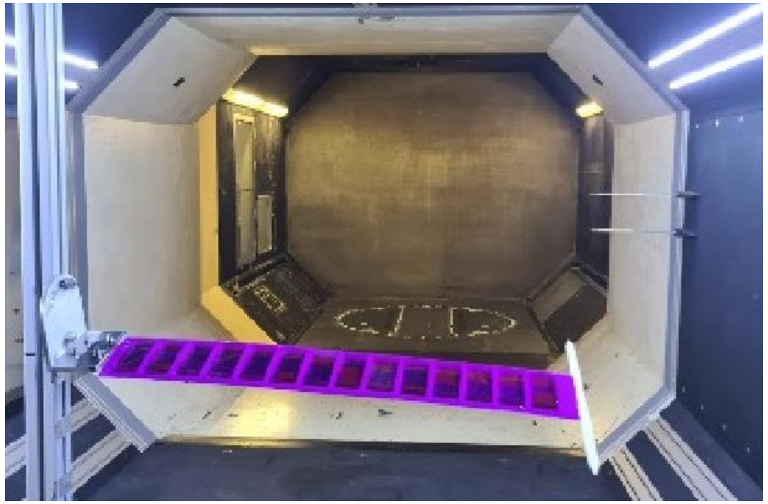
ADiGator

A Source Transformation via Operator Overloading Toolbox for the Automatic Differentiation of Mathematical Functions in MATLAB

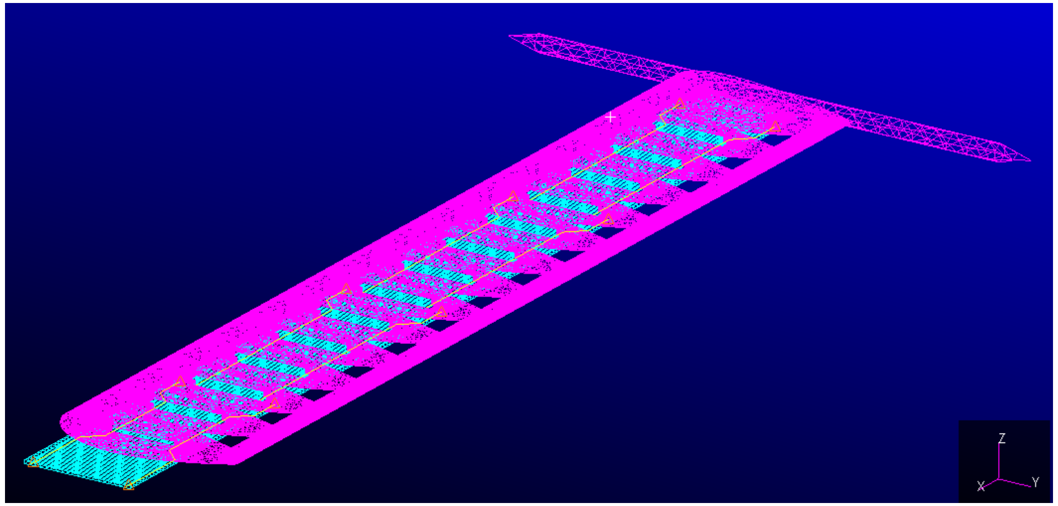
Table of Contents

Aeroelastic test-case

- Pazy wing
  - **Highly flexible** wing geometry for the Aeroelastic Prediction Workshop (AePW): $L_{span} = 0.550\ m$, $c = 0.100\ m$



The Sapienza Pazy wing in the SOTON 7x5 wind tunnel [REF]



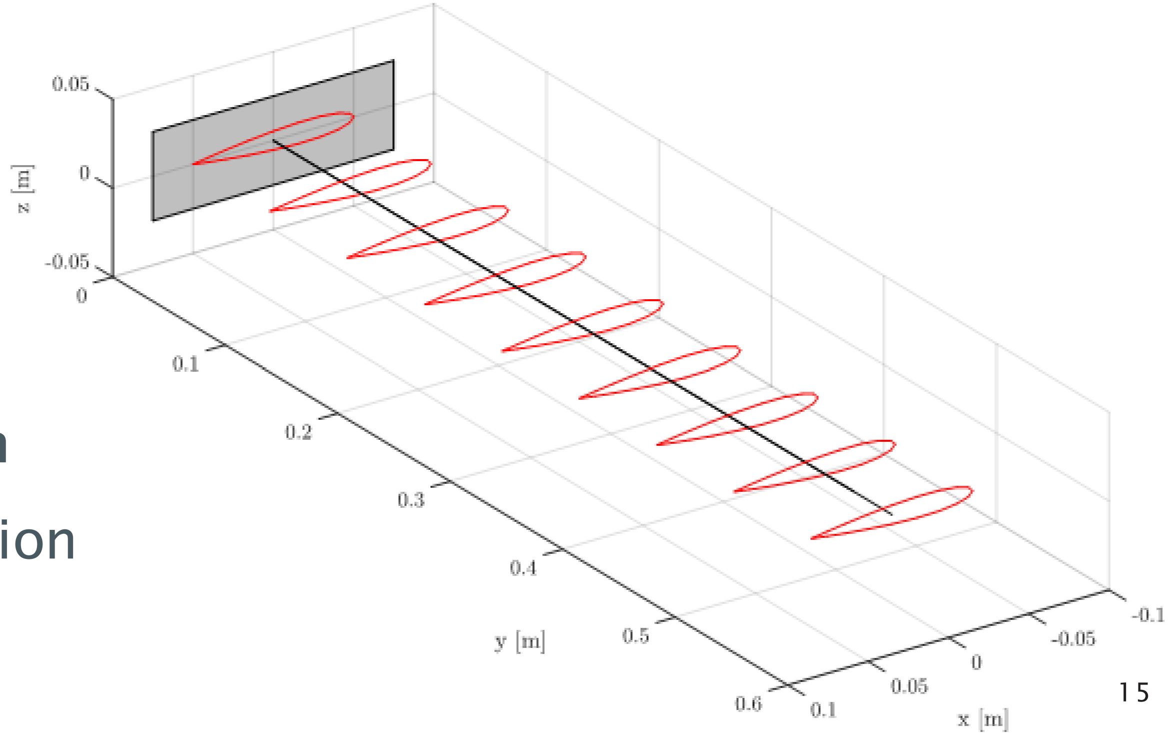
Sapienza's Pazy Nastran 3D FEM

Proof-of-concept Pazy wing modelling

- Structural dynamic modelling
  - Hodges' **fully intrinsic** geometrically exact **nonlinear beam**
    - Assumption of **constant spanwise stiffness and inertial properties**
- Unsteady aerodynamic modelling
  - Peters' **2D finite-state** unsteady aerodynamics with a **tip-loss correction**
    - Tip-loss correction from a **VLM** solution around the undeformed geometry

Mode Frequency (Hz)

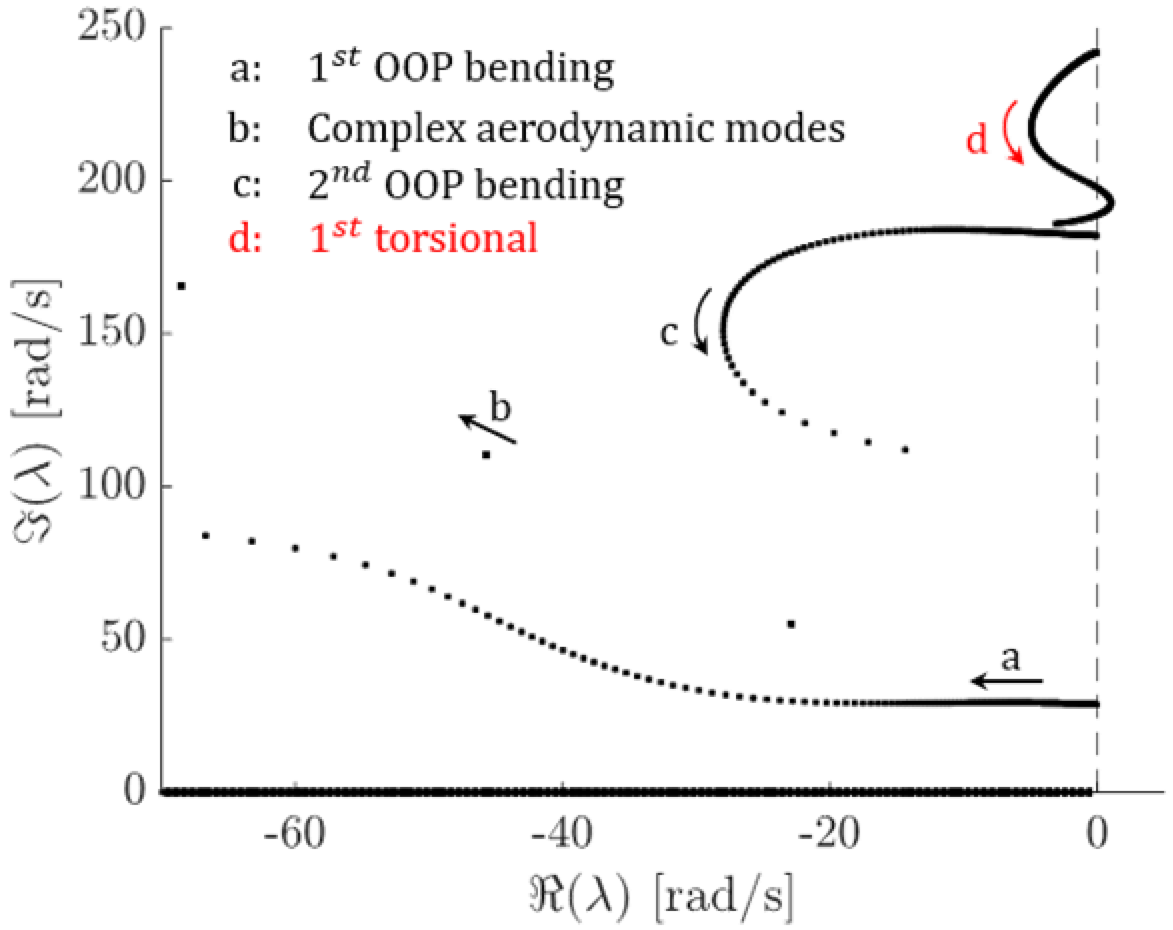
Mode	Coppotelli (2025)	Present
First OOP bending	4.15	4.59
Second OOP bending	25.50	29.17
First torsional	38.45	38.89



Aeroelastic Full Order Model

Flight conditions	
$\alpha_\infty =$	$0.0°$
$\rho_\infty =$	$1.225\ kgm^{-3}$

- Nonlinear system of ODEs, $\dot{X} = f(X)$
  - **16 beam elements**
- Total of 288 aeroelastic states
  - 192 structural states
  - 96 aerodynamic states
- **Hump mode** flutter phenomenon
  - Flutter onset: $52.2\ ms^{-1}$
  - Flutter offset: $62.8\ ms^{-1}$



Limit-cycle oscillation study

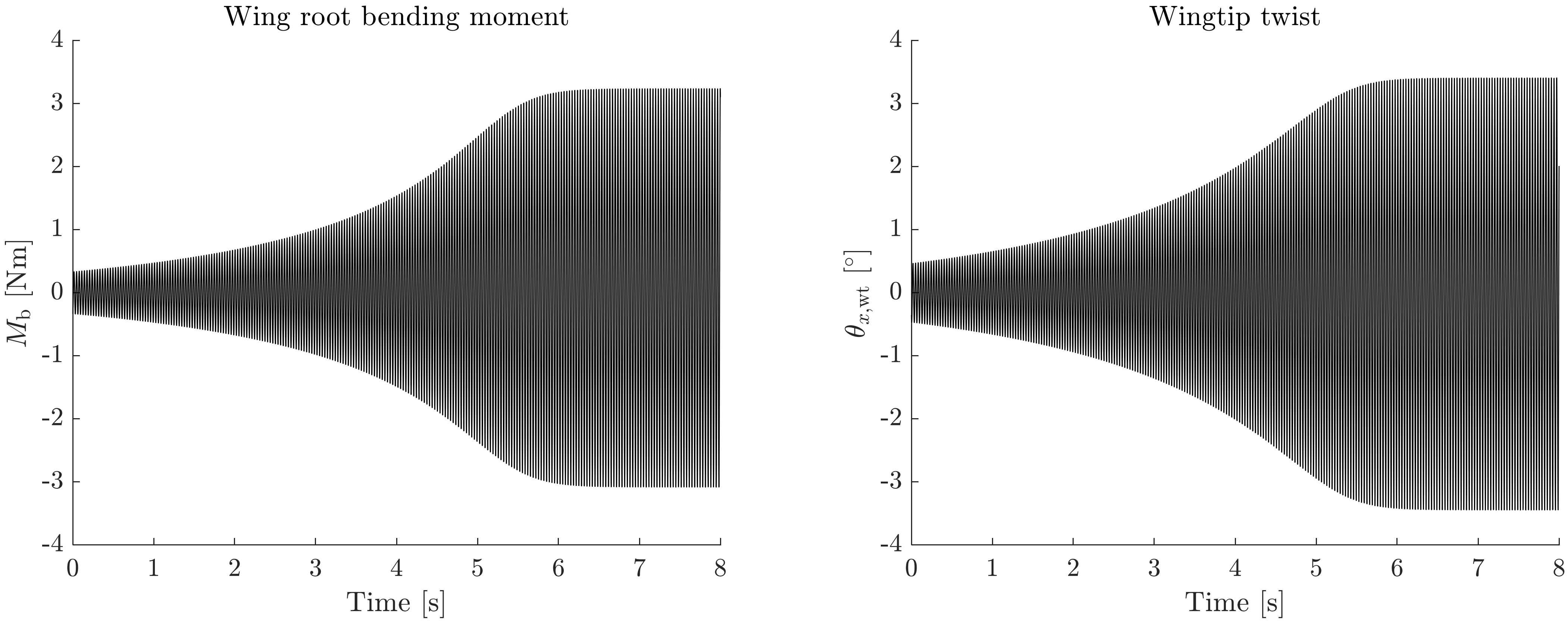
- **Supercritical** LCO analysis
  - Onwards from flutter onset, $V_f = 52.2\, ms^{-1}$

- Dynamics excited by a perturbation in the initial conditions
  - Perturbed by an amount of the aeroelastic critical mode

Full model limit cycles

Flight conditions	
$\alpha_\infty =$	$0.0°$
$\rho_\infty =$	$1.225\ kgm^{-3}$

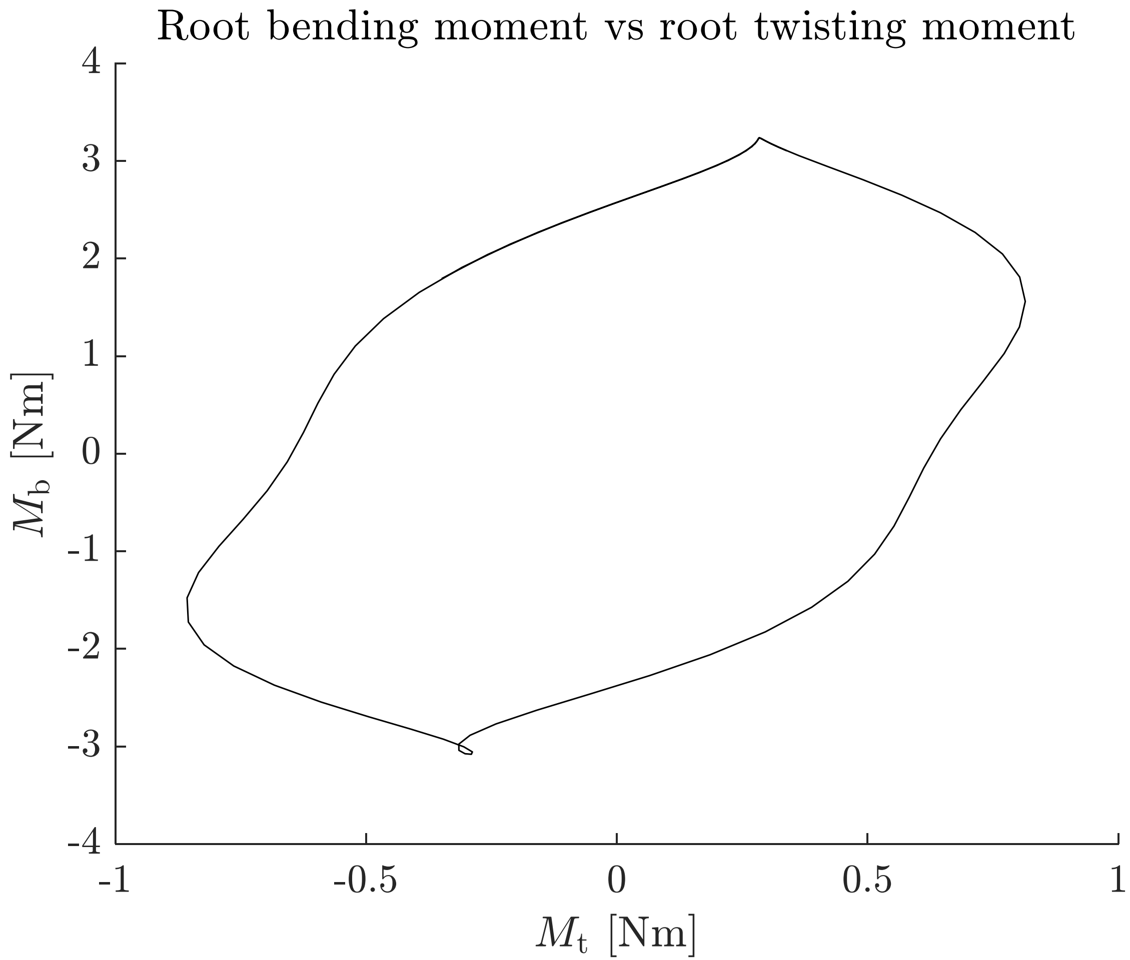
- LCO dynamics at 3% past the flutter speed



Full model limit cycles

- LCO phase portrait at 3% past the flutter speed
- Wing root bending moment vs wing root torque phase plot
- Transients omitted (only the outermost LCO loop plotted)

Flight conditions	
$\alpha_\infty =$	$0.0°$
$\rho_\infty =$	$1.225\ kgm^{-3}$



University of Southampton

Table of Contents

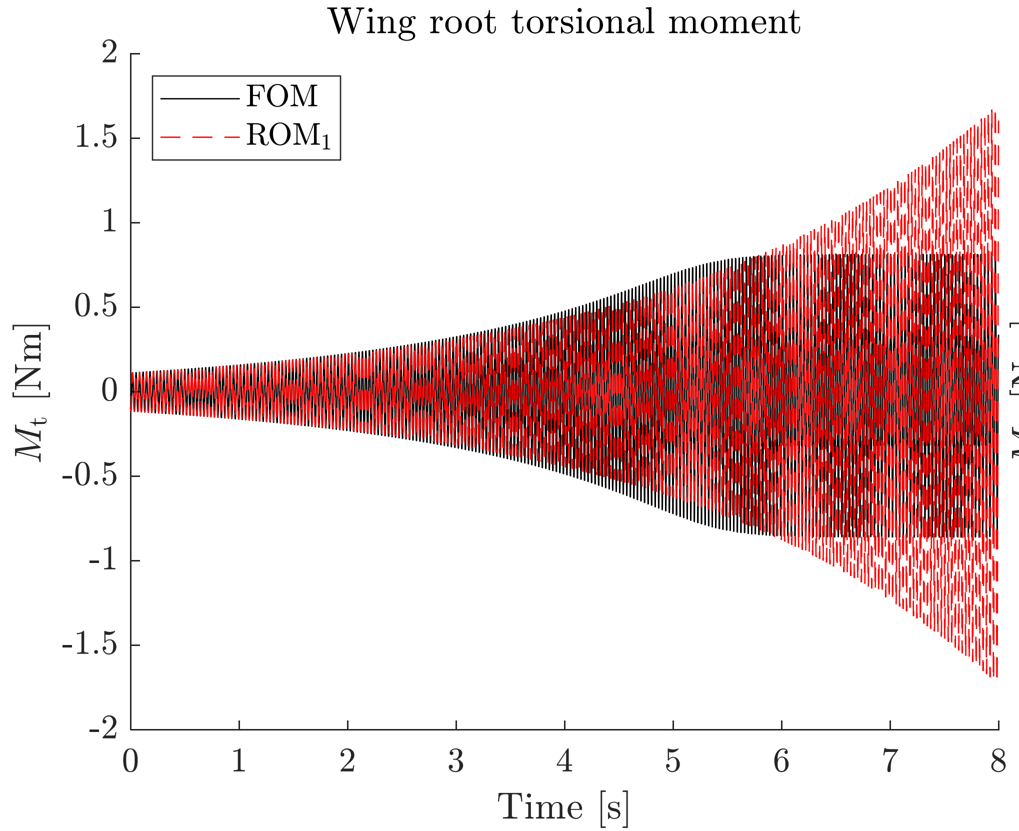
Reduced order modelling

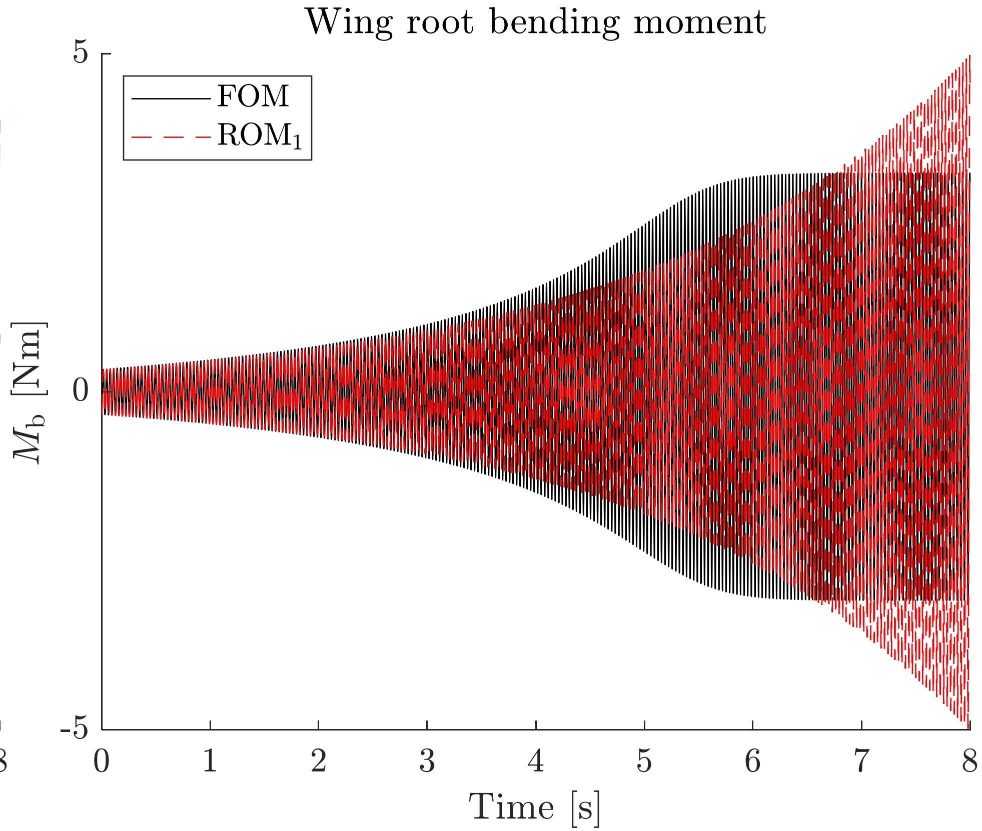
- Create ROMs to predict LCO dynamics
  - Retain up to **quadratic nonlinearities** in the **reduced** Taylor expansion

- Nonlinear ROMs with increasing **aeroelastic** modal basis size
  - Begin with a **1 mode projection**

Reduced order model

- 1 mode ROM
  - 1st **torsional** (critical mode)

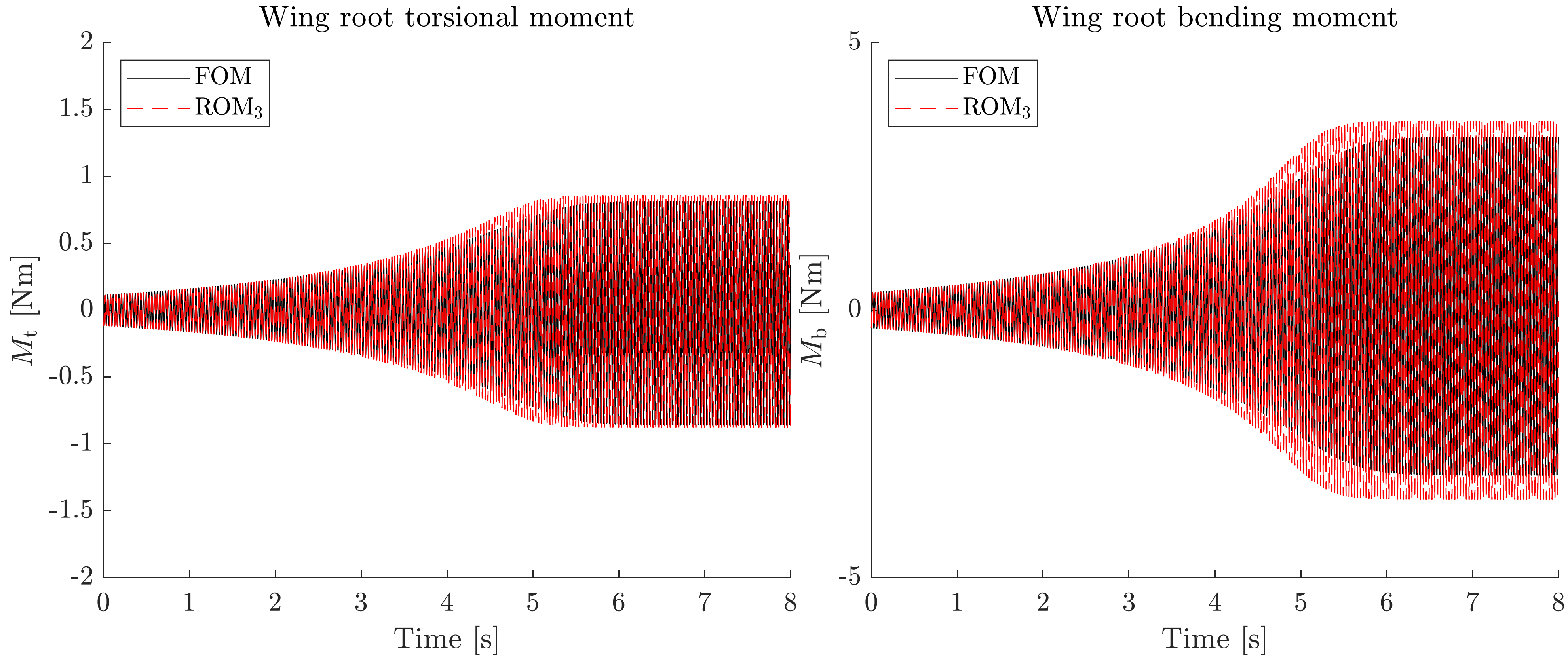




Acronyms	
OOP	Out-of-plane
IP	In-plane

Reduced order model

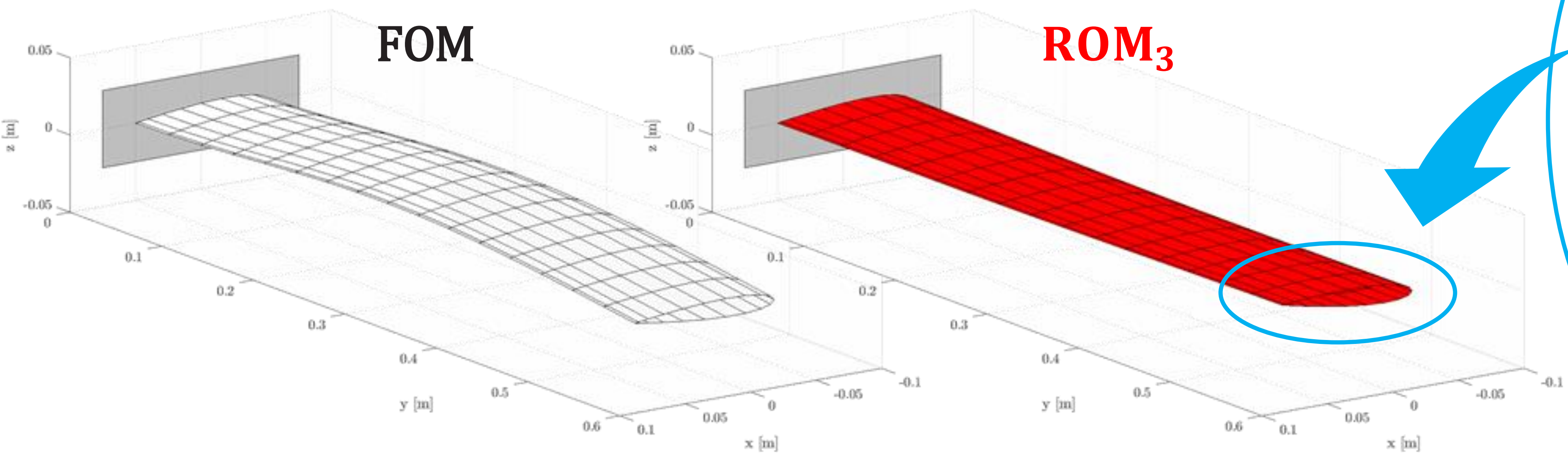
- 3 mode ROM
  - **+ 1 OOP bending + 1 IP bending**

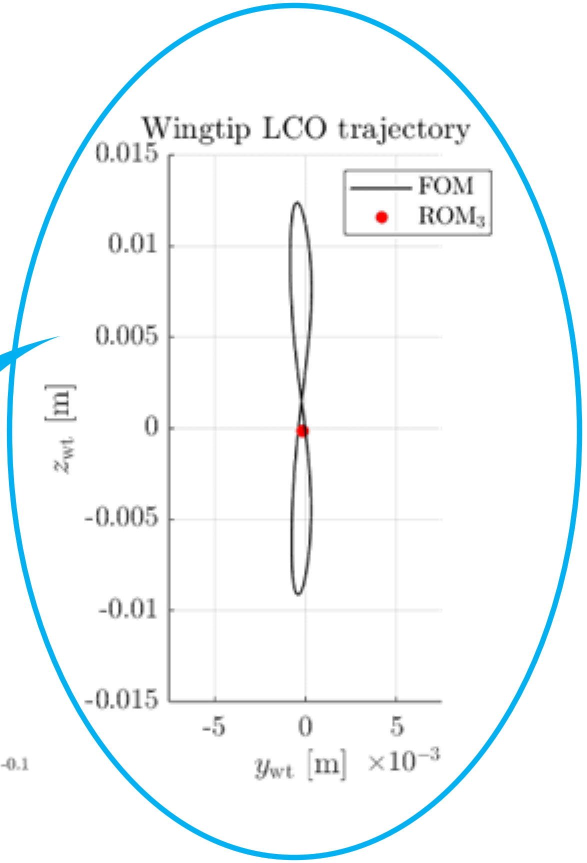


Acronyms	
OOP	Out-of-plane
IP	In-plane

Reduced order model

- 3 mode ROM
  - + 1 **OOP bending** + 1 **IP bending**



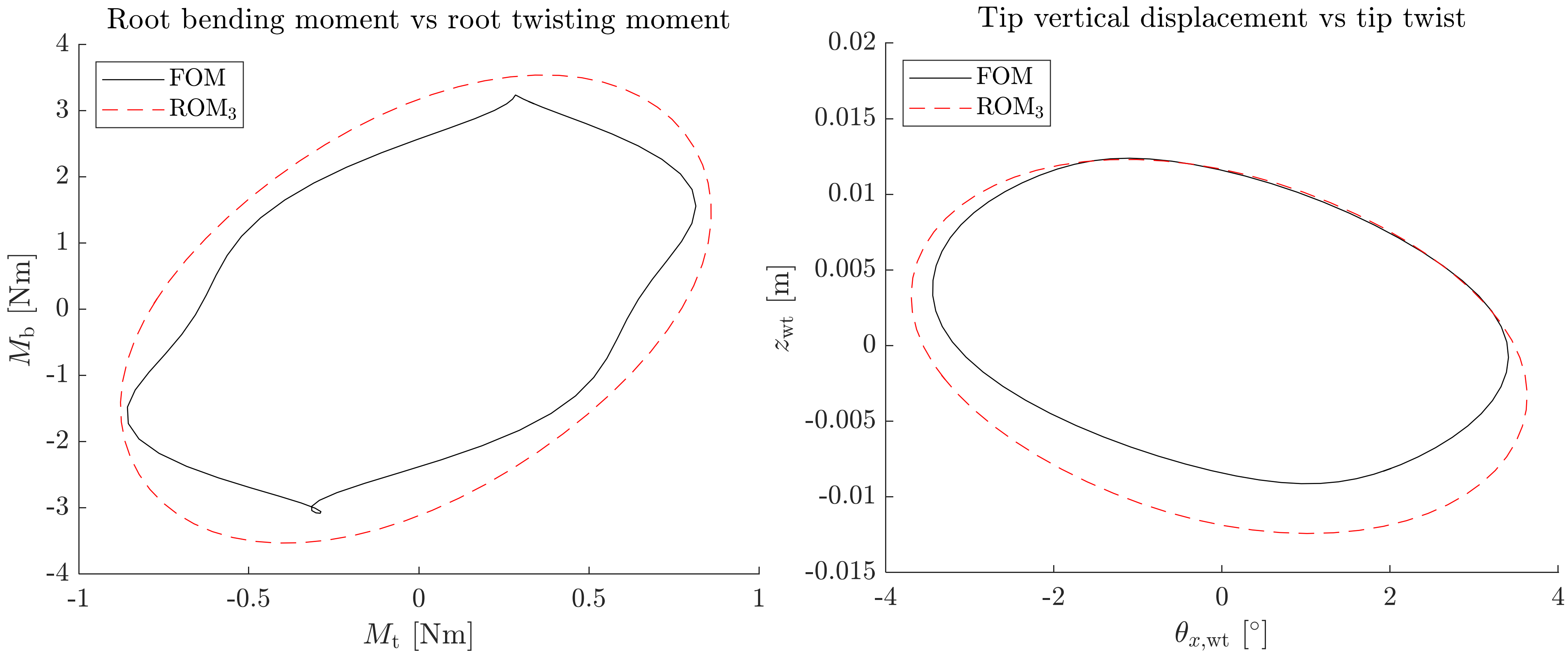


Animations at 0.03 × real-time

Acronyms	
OOP	Out-of-plane
IP	In-plane

Reduced order model

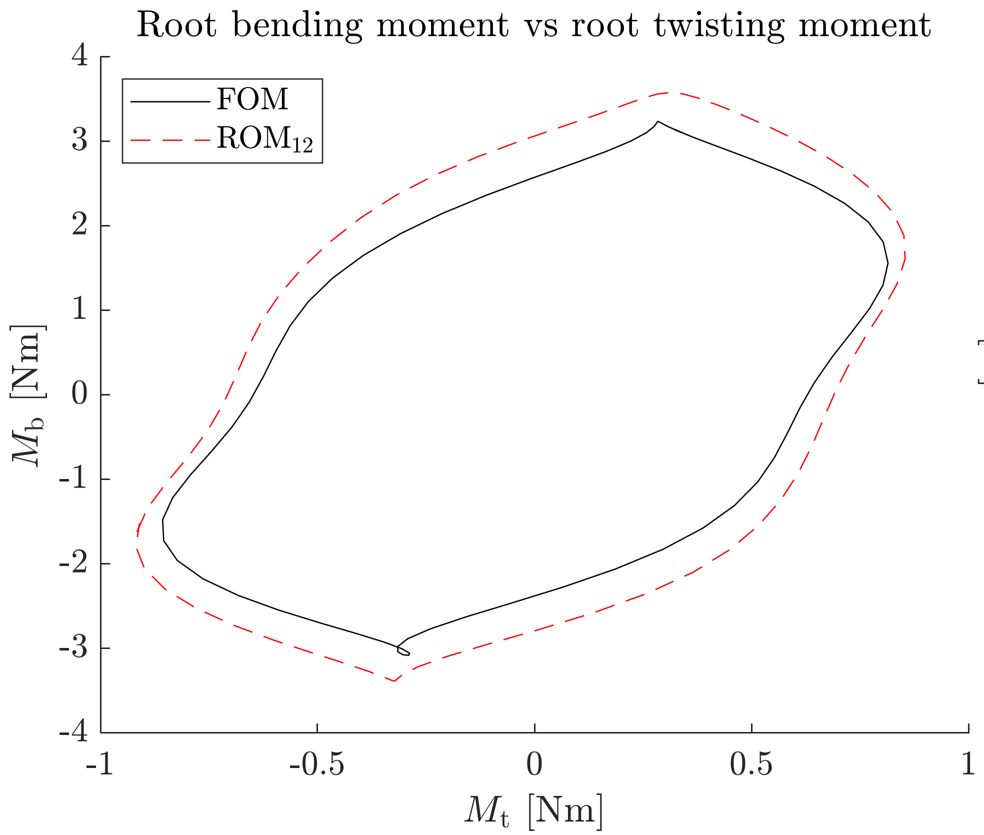
- 3 mode ROM
  - **+ 1 OOP bending + 1 IP bending**

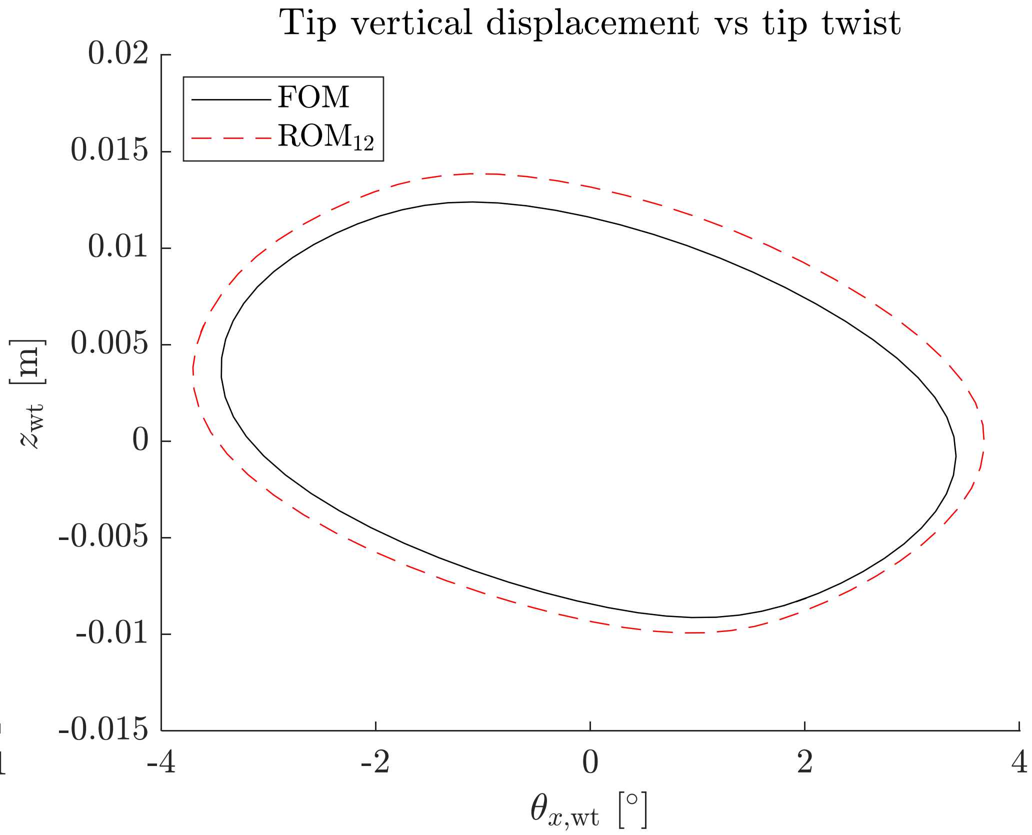


Acronyms	
OOP	Out-of-plane
IP	In-plane

Reduced order model

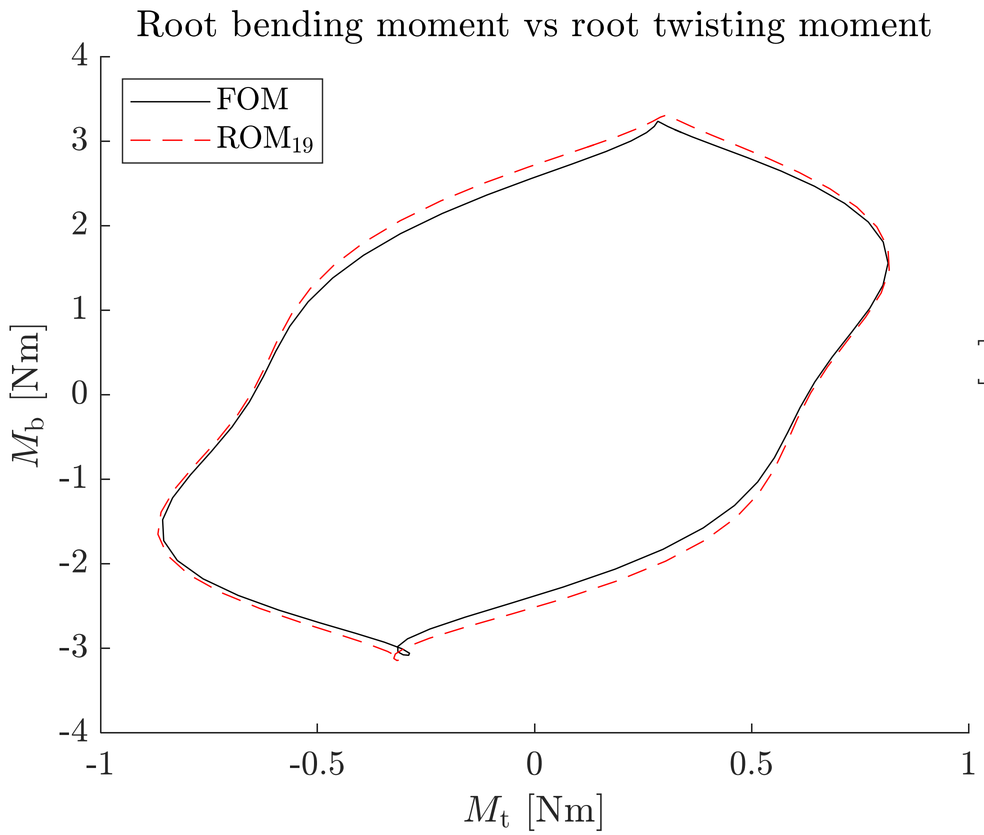
- 12 mode ROM
  - + 1 **aero mode** + 4 **OOP bending** modes + 4 **torsional** modes

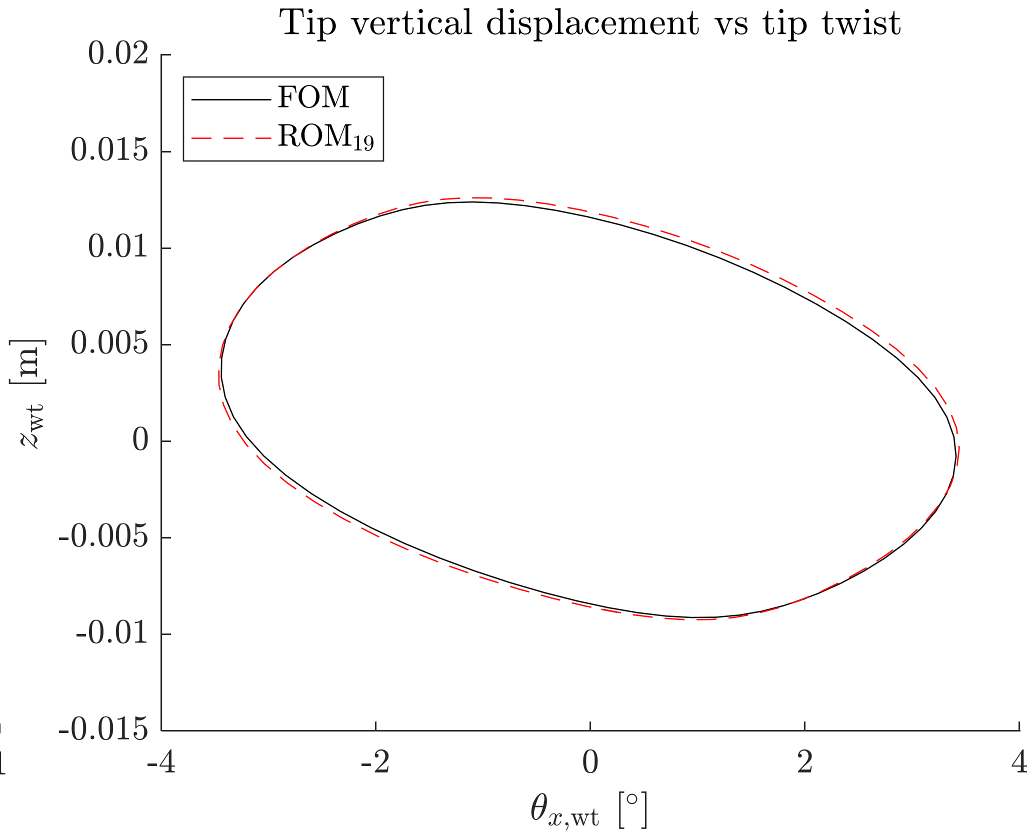




Reduced order model

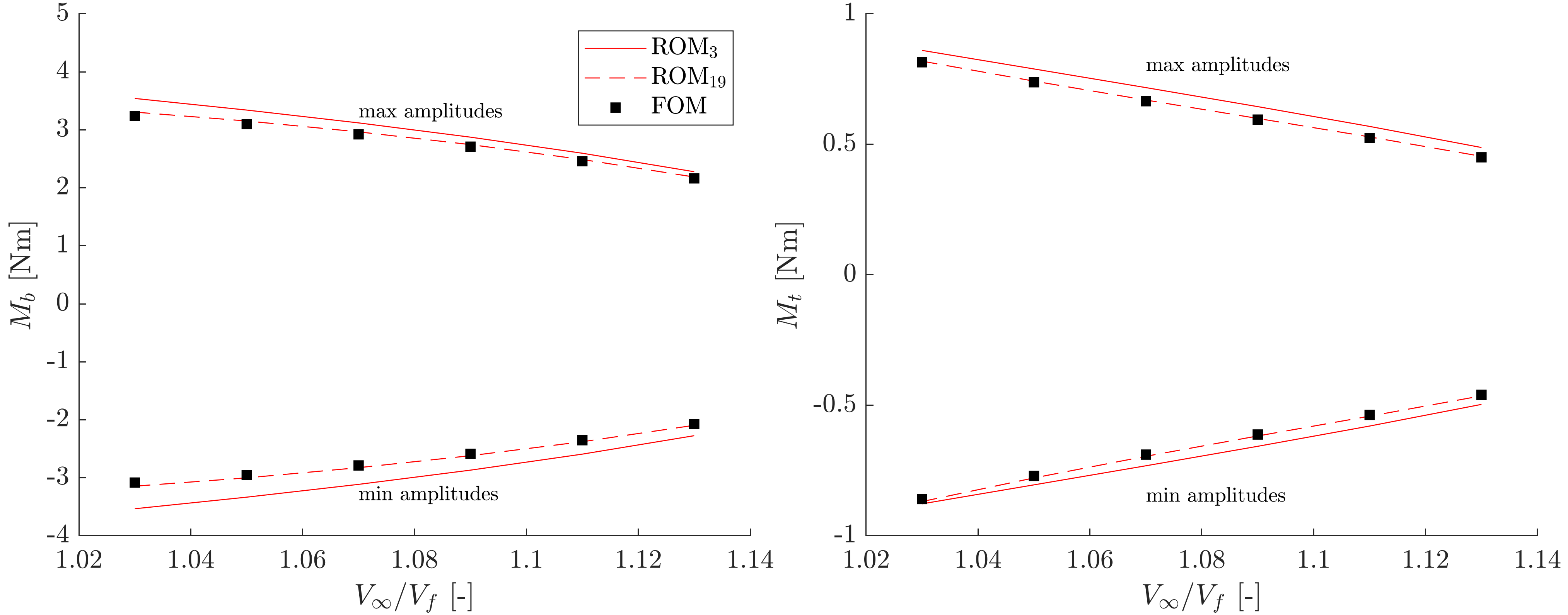
- 19 mode ROM
  - + 1 **IP bending** + 2 **OOP bending** + 4 **torsional**





Limit cycle amplitude envelope

- LCO amplitude over increasing flutter speed ratio, V_∞ / V_f
  - 3 mode ROM provides conservative approximation



Computational cost

- "Offline"
  - **ONE-OFF** cost to source-transform AD ROM codes
- "Online AD"
  - Cost to evaluate AD ROM code
- "Online dyn. resp"
  - Cost to compute **8 seconds** of dynamic response

	Integration scheme	Step size [s]
FOM	Newmark-Beta	0.0005
ROM_3	RK4	0.0005
ROM_{19}	Rk4	0.0005

	Offline [s]	Online AD [s]	Online dyn. resp [s]
FOM	–	–	603.17
ROM_3	39.40	0.09	2.49
ROM_{19}	40.23	0.15	6.48

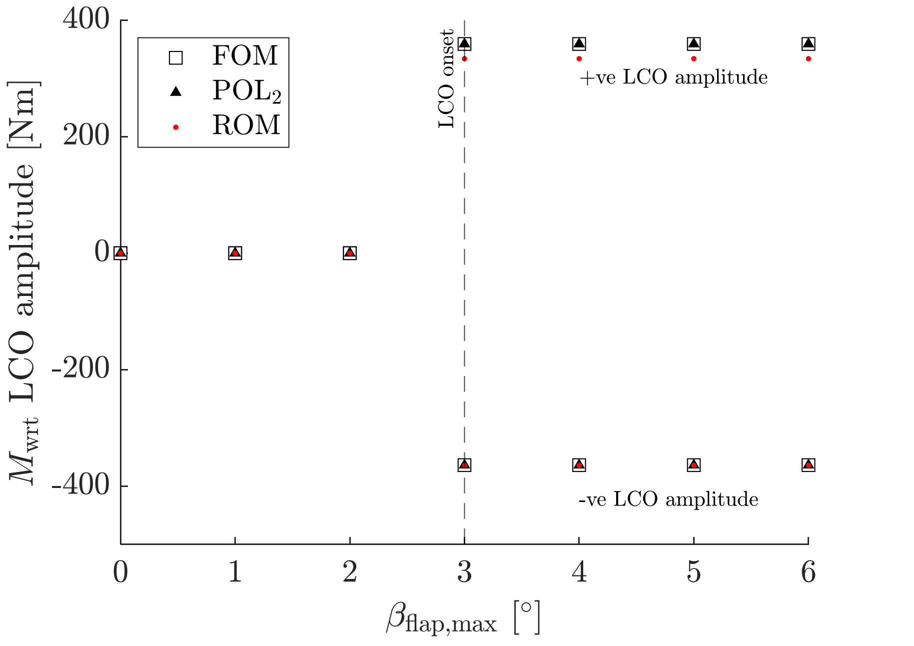
Conclusions

- Projection nonlinear ROM capable of capturing **dynamic nonlinearity** with a **small subset of aeroelastic modes**
- Nonlinear ROMs can capture increasing levels of nonlinear dynamic fidelity by careful addition of modes
  - **3 mode** nonlinear ROM to capture **LCO amplitudes**
  - **19 mode** nonlinear ROM to capture **LCO trajectory**
- ROMs evaluated at a fraction of the cost of the FOM and **DO NOT** require FOM dynamic responses for ROM construction

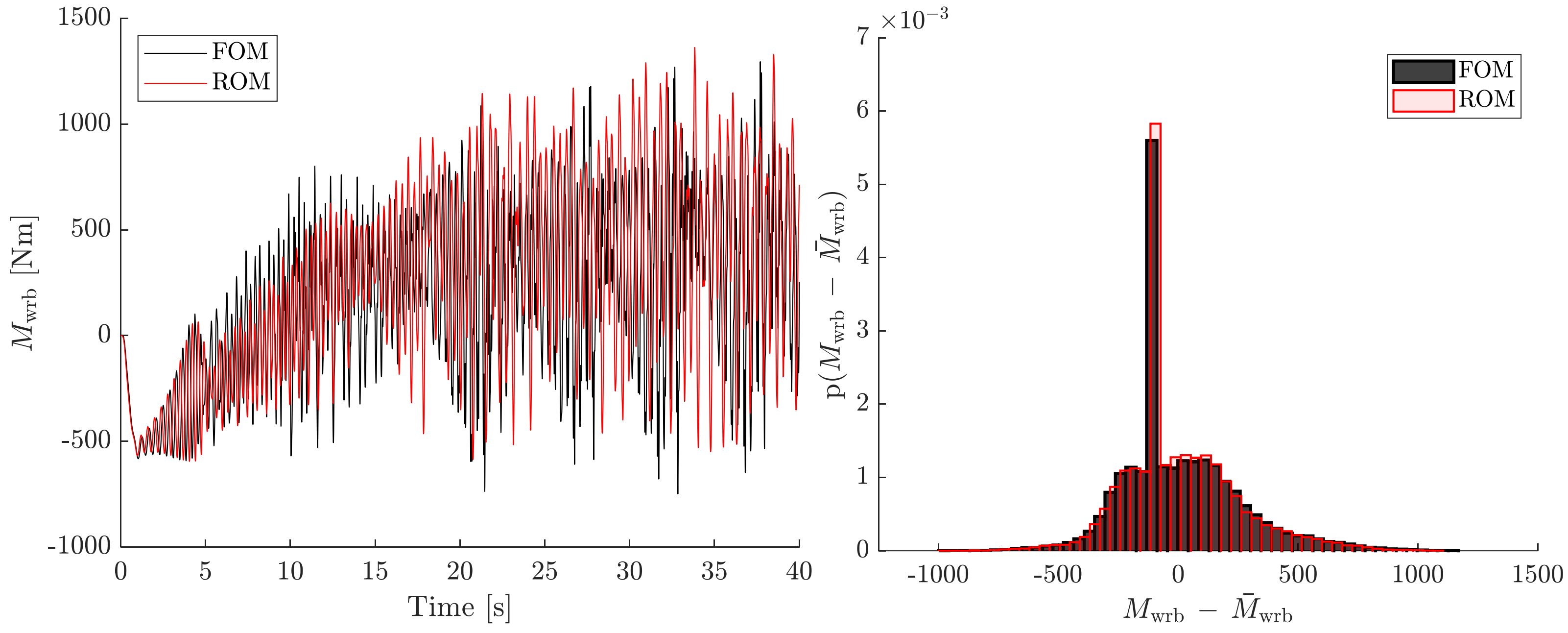
Future work

- Ongoing research with the high aspect ratio Patil wing, $AR = 16$

Subcritical LCO study (flap excitation)



Recovering statistical properties of aeroelastic chaos



AIAA Online Short Course

- “Machine Learning for Aircraft Applications”
  - Special emphasis on ROMs
  - 33 hours of ROM fun
  - From 08 SEP 2025 – 13 OCT 2025

Instructors

Dr David Massegur

Professor Andrea Da Ronch

Me

ONLINE SHORT COURSE

MACHINE LEARNING FOR AIRCRAFT APPLICATIONS

*Instructed by experts from the University of Southampton (United Kingdom): **Dr. Andrea Da Ronch**, Professor of Aeronautics and Astronautics; **Dr. David Massegur**, Senior Research Assistant, and Engineering and Machine Learning Consultant, DIMAE Technologies; and **Mr. Declan Clifford**, Senior Research Assistant, aeroelasticity researcher for the Airbus OneHeart Project.*

OVERVIEW

This course is designed to provide a comprehensive introduction to the application of reduced-order model (ROM) techniques in aerospace engineering, focusing on aircraft performance and aerodynamic load analysis. The ROM techniques are classified into two categories: data-driven methods based on machine learning (ML); and equations-derived methods based on nonlinear projections (NP). The course aims to equip students with a foundational understanding of ML and NP concepts and practical skills for implementing these approaches in aerospace contexts. By the end of the course, participants will gain knowledge of cutting-edge techniques and their relevance to complex aerospace problems, particularly in predicting steady and unsteady aerodynamic loads, and performing aeroelastic analysis.

AUDIENCE

This course is designed for students and researchers in academia and industry who are focused on advanced topics in aerospace engineering, particularly in aerodynamic load prediction and reduced-order modeling. It also caters to technical decision-makers who seek to understand emerging machine learning and projection-based techniques for future development strategies. The content is tailored to equip participants with both foundational knowledge and practical skills, ensuring they can apply modern machine learning and reduced-order model techniques to real-world aerospace challenges and make informed decisions in research and industrial settings.

LEARNING OBJECTIVES

- Define key aerospace engineering problems related to aircraft development.
- Recognize the complexities in predicting steady-state and unsteady aerodynamic loads.
- Explain the use of ROMs in complex aerospace computational problems.
- Understand the foundational concepts of ML and NP and their relevance to aerospace engineering.
- Differentiate between data-driven ROMs and equations-derived ROMs.
- Discuss the application of ML techniques like autoencoders and graph neural networks to develop data-driven ROMs.
- Discuss implementation aspects related to projection-based ROMs.
- Assess the impact of integrating ROM techniques in real-world aerospace applications, demonstrating practical problem-solving skills.
- Critically assess the benefits and limitations of using ML and projection methods in aerospace contexts for aircraft load analysis.

DETAILS

DATES: From 8 September – 13 October 2025 (5.5 Weeks, 11 Classes, 33 Total Hours)

TIME: Every Monday and Wednesday at 12–3 p.m. Eastern Time (UTC-5)

COST: AIAA Member Price: $1295 USD / Non-Member Price: $1495 USD / AIAA Student Member Price: $695 USD

All sessions will be recorded and available for on-demand replay; course notes will be available for download.

REGISTER NOW

learning.aiaa.org

AIAA

SHAPING THE FUTURE OF AEROSPACE

Acknowledgements

- Giuliano Coppotelli & the Sapienza team
  - For Pazy wing modelling support

- Marcello Righi
  - For Pazy wing modelling support

- This work was supported by Innovate UK [Grant number 10003388].
- We also acknowledge support from AIRBUS UK.