Assessing Validity of Mesh Refinement Sequences

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Overview

Given: two computational mesh

- On each mesh, compute:
  - Cell size (length scale)
  - Anisotropy
  - Orientation
- Project data from one mesh to the other and compare

Begin with Cell Second Moments

These moments can be written as, for example,

$$
\begin{aligned}
I_{xx} &= \frac{1}{S_i} \int_i x^2 \, dS \\
I_{xz} &= \frac{1}{S_i} \int_i xz \, dS
\end{aligned}
$$

where integral is over area in 2D and over volume in 3D.
Can also evaluate moments by applying Gauss's Theorem and integrating around the cell. For instance,

$$
\begin{aligned}
I_{xx} &= \frac{1}{S_i} \int_{\partial i} \frac{x^3}{3} n_x \, dA \\
I_{xz} &= \frac{1}{S_i} \int_{\partial i} \frac{x^2 z}{2} n_x \, dA
\end{aligned}
$$

Aspect Ratio and Anisotropy

Two Dimensions

- Moment of inertia tensor is

$$M = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

- From eigenvalues or Mohr's circle,

$$\begin{aligned} I_{\text{max}} &= \frac{I_{xx} + I_{yy} + \sqrt{(I_{xx} - I_{yy})^2 + 4I_{xy}^2}}{2} \\ I_{\text{min}} &= \frac{I_{xx} + I_{yy} - \sqrt{(I_{xx} - I_{yy})^2 + 4I_{xy}^2}}{2} \\ \theta &= \frac{1}{2} \arctan\left(\frac{2I_{xy}}{I_{xx} - I_{yy}}\right) \end{aligned}$$

- Aspect ratio:

$$AR = \sqrt{\frac{I_{\text{max}}}{I_{\text{min}}}}$$

Aspect Ratio and Anisotropy

Three Dimensions

- Moment of inertia tensor is:

$$M = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

- Find eigenvalues and eigenvectors numerically. Then:

$$\begin{aligned} AR_1 &= \sqrt{\frac{\lambda_{\text{max}}}{\lambda_{\text{med}}}} \\ AR_2 &= \sqrt{\frac{\lambda_{\text{med}}}{\lambda_{\text{min}}}} \end{aligned}$$

- Orientation directly from eigenvectors.

Projecting Data Between Meshes

- Read size and coordinates (currently: VGrid and UGrid)
- Partition meshes using unbalanced recursive bisection.
- For each mesh, write partitioned files with connectivity.
- For each part,
  - Read fine and coarse mesh data
  - Create cell-to-cell connectivity and geometric search tree for fine mesh
  - For each coarse mesh cell,
    - Find fine cell containing centroid
    - Use injection to get fine mesh data onto coarse mesh
    - Write comparison data into global array
- Write comparison data file for visualization.

Comparison Between Meshes

Length scale. We expect that, locally,

$$\frac{L_2}{L_1} = \sqrt[d]{\frac{N_1}{N_2}}$$

where N is vertex count. So compute and compare refine size quality:

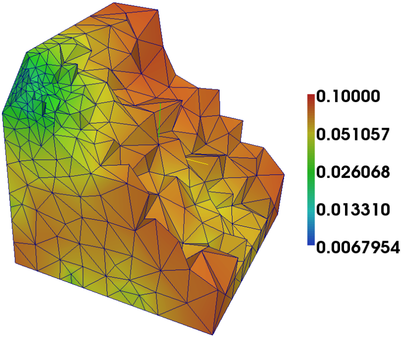
$$Q_r \equiv \frac{L_1}{L_2} \sqrt[d]{\frac{N_1}{N_2}} \quad (1)$$

Aspect ratio. Ratio of aspect ratios should be one.

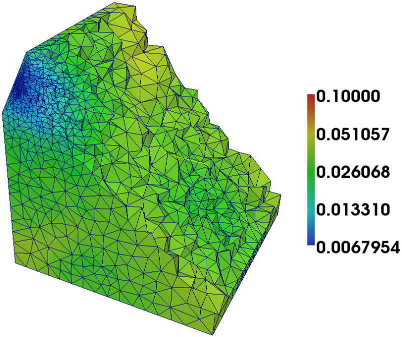
Orientation. Principal directions should align.

Isotropic Mesh in a Clipped Cube

Cell Size



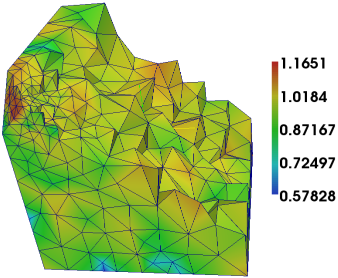
Coarse mesh (5564 cells)



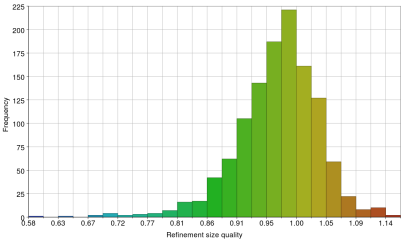
Fine mesh (41826 cells)

Isotropic Mesh in a Clipped Cube

Refinement Size Quality Q_r



Spatial Distribution



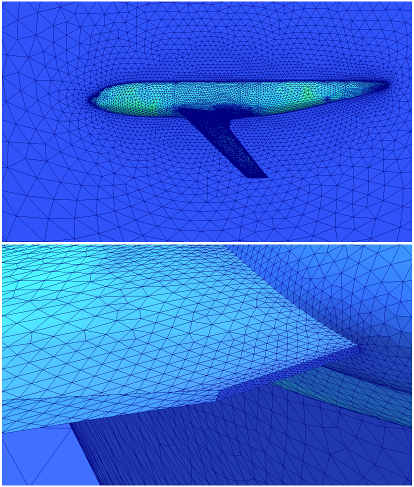
Histogram

DPW-III Mesh Pair

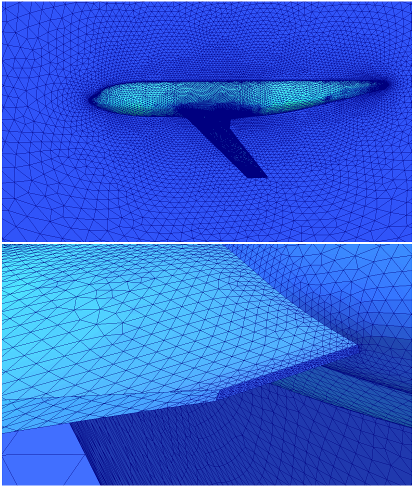
- DLR-F6 FX2B wing-body geometry
- Raytheon “coarse” and “medium” meshes
  - All tetrahedral
  - Very coarse for RANS simulations
    - Coarse mesh: 544K vertices, 3.14M tetrahedra
    - Fine mesh: 1.08M vertices, 6.28M tetrahedra
  - Surface mesh isotropic except near leading and trailing edges
    - So only one aspect ratio and orientation matter

DPW-III Mesh Pair

Coarse mesh

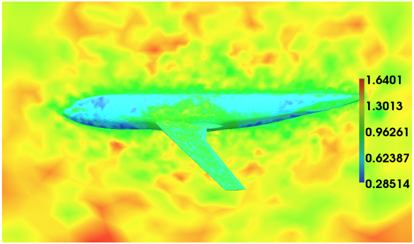


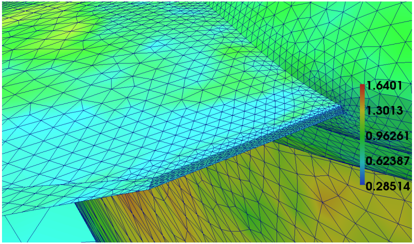
Fine mesh

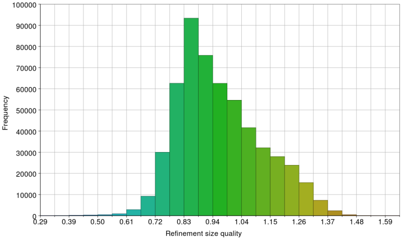


DPW-III Mesh Pair

Refinement Size Quality





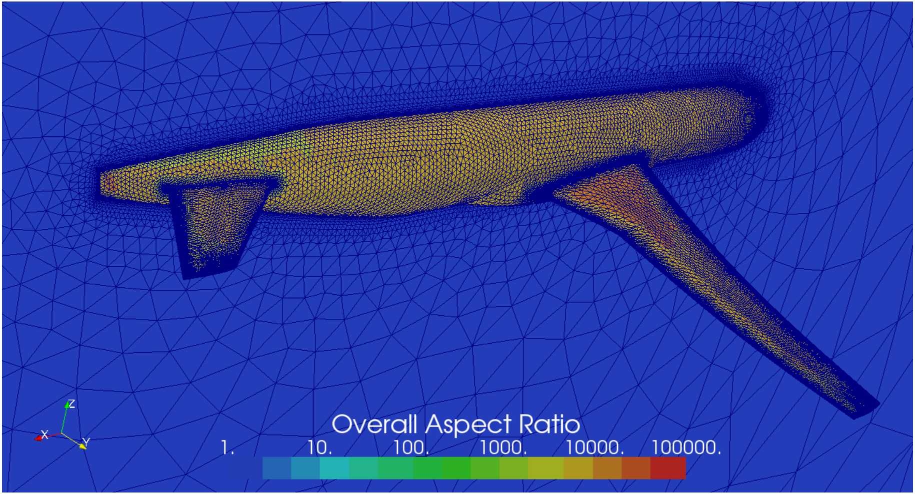


Meshes Considered

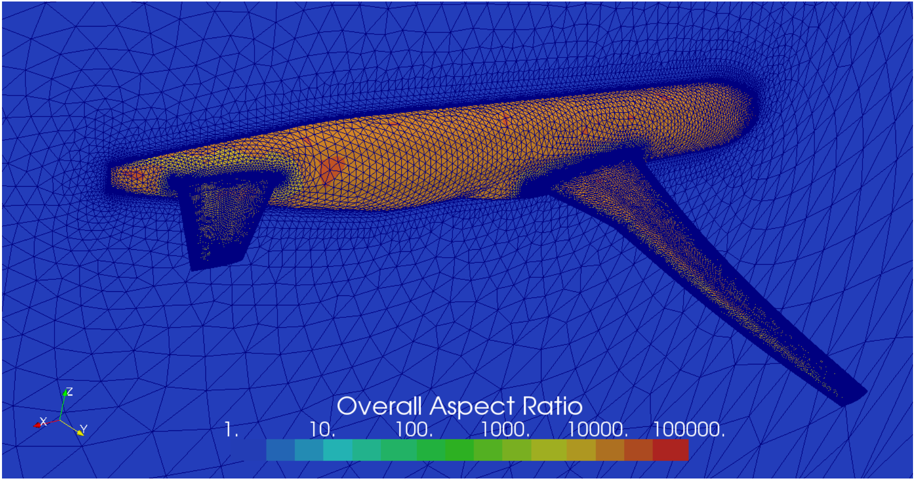
Only 0-degree tail cases

Source	Size	# Verts	# Tets	# Pyrs	# Prisms
Langley (CC)	C	672K	3.94M	—	—
	M	1.71M	10.0M	—	—
	F	5.92M	34.9M	—	—
Langley (VC)	C	3.66M	21.6M	—	—
	M	10.2M	60.3M	—	—
Cessna	C	3.55M	20.8M	—	—
	M	9.93M	58.2M	—	—
Boeing	C	2.43M	2.26M	48.7K	3.88M
	M	9.45M	4.45M	169K	16.9M
	F	20.7M	21.7M	332K	33.4M
Boeing (best practice)	C	4.00M	5.52M	44.5K	5.88M
	M	18.3M	8.13M	229K	33.3M

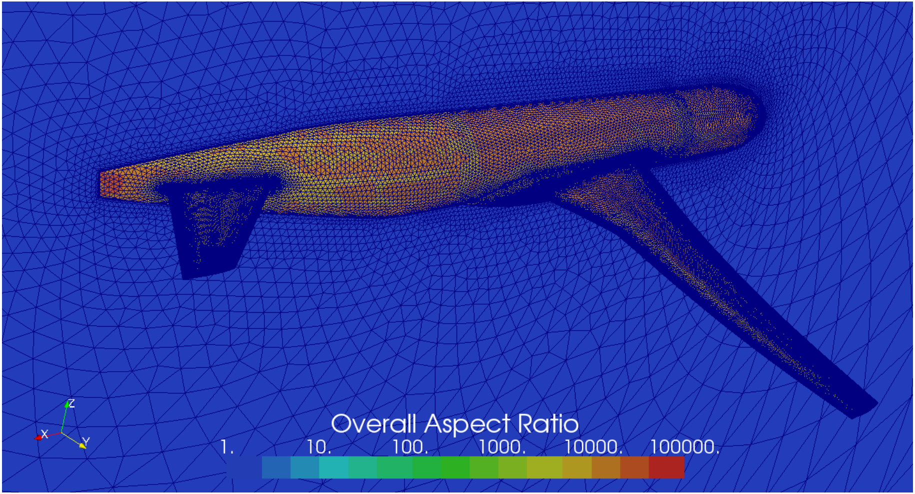
Langley Medium Cell-Centered Mesh



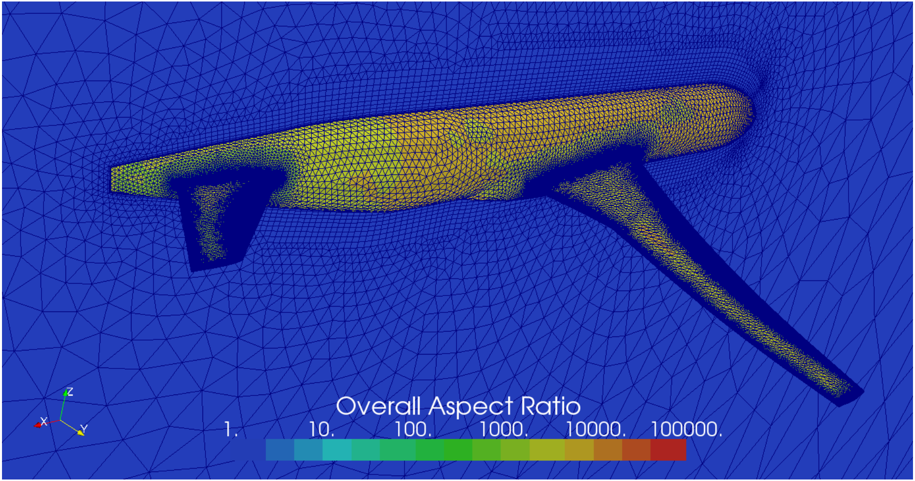
Langley Coarse Vertex-Centered Mesh



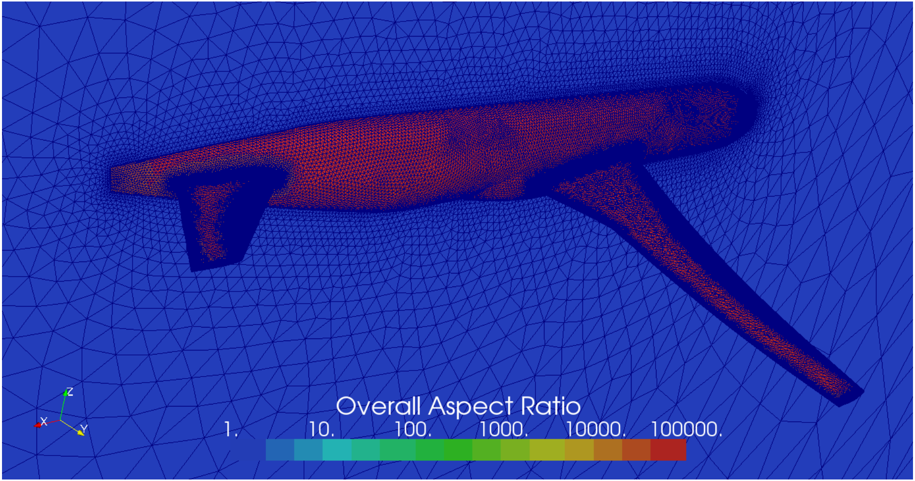
Cessna Coarse Mesh



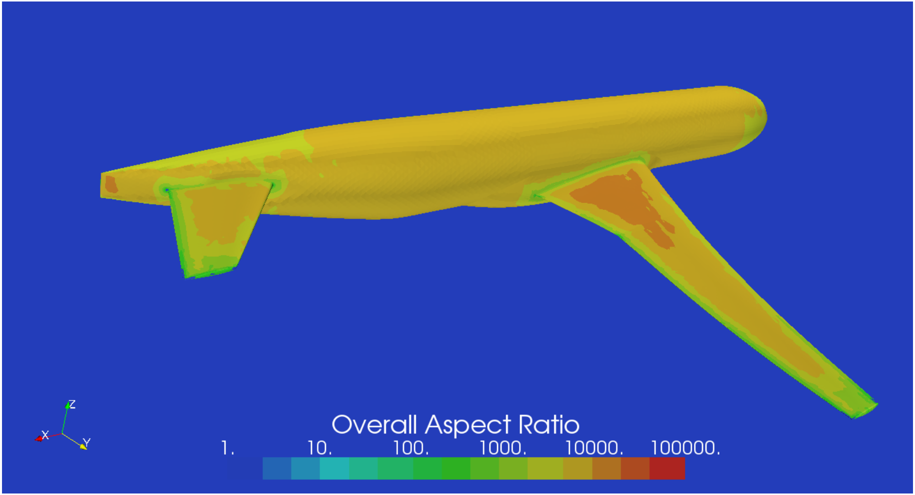
Boeing Coarse Mesh



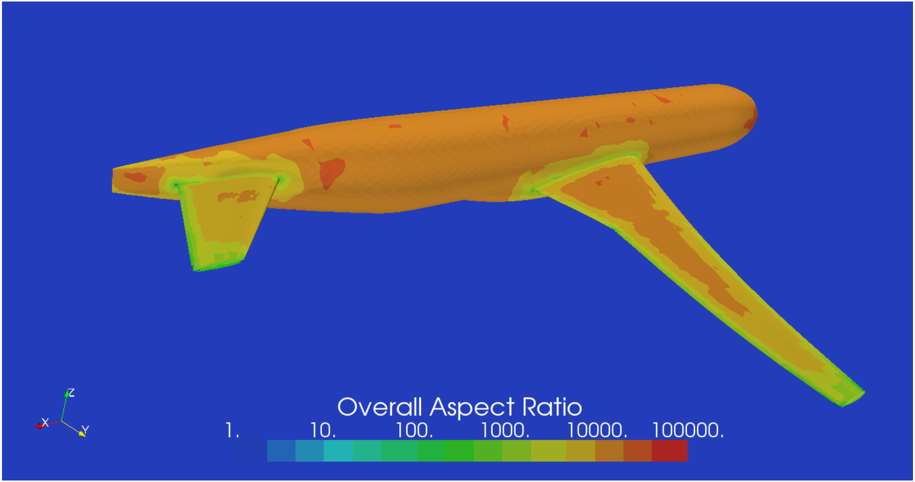
Boeing Best Practice Coarse Mesh



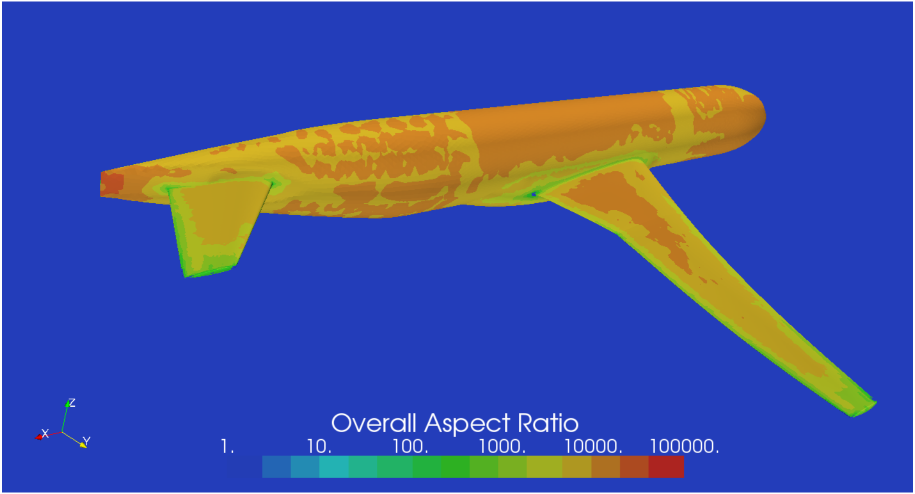
Langley Medium Mesh



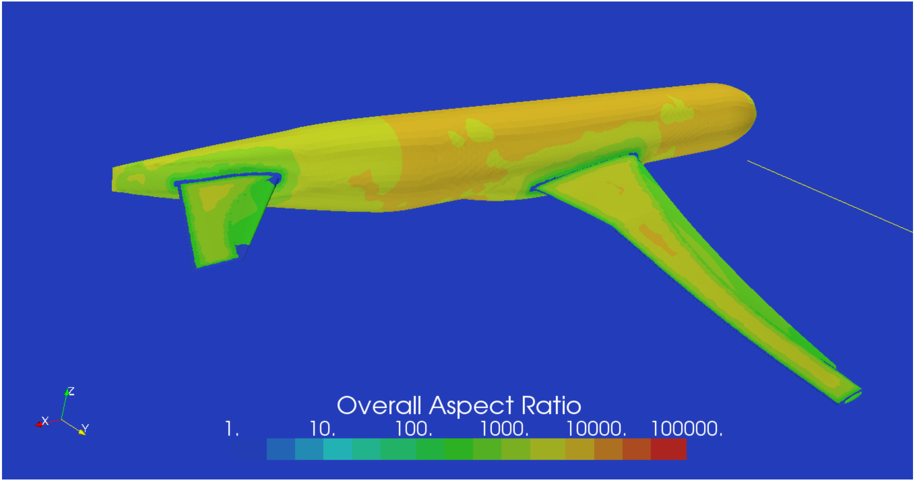
Langley Coarse Vertex-Centered Mesh



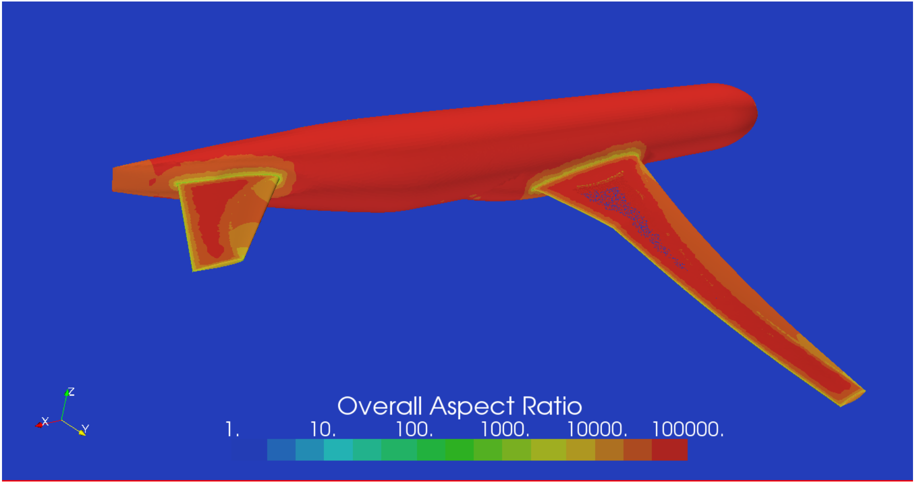
Cessna Coarse Mesh



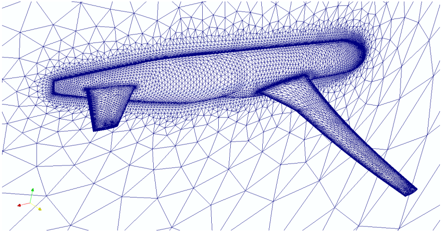
Boeing Coarse Mesh

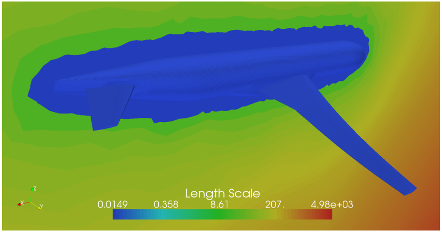


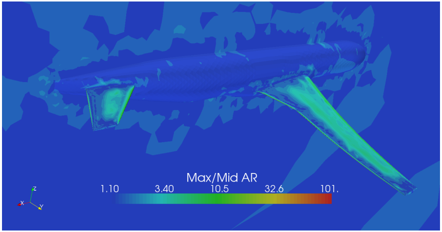
Boeing Best Practice Coarse Mesh

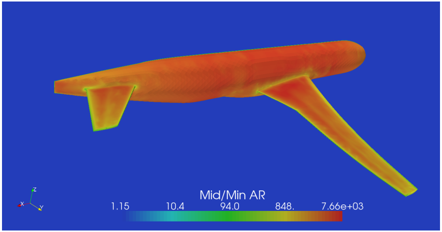


Langley Cell-Centered Coarse Mesh

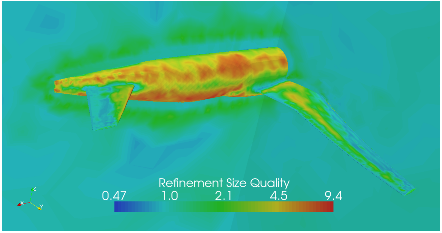


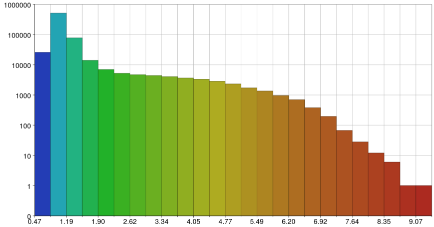


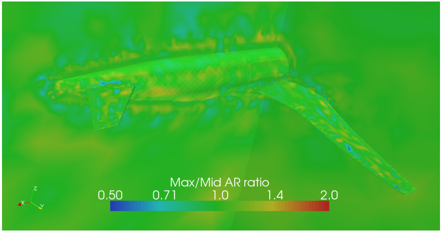


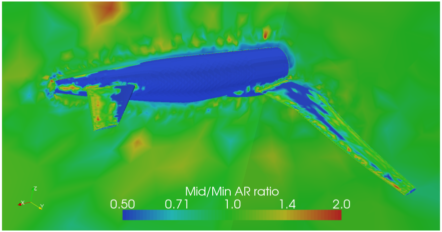


Langley CC Coarse-Medium Comparison

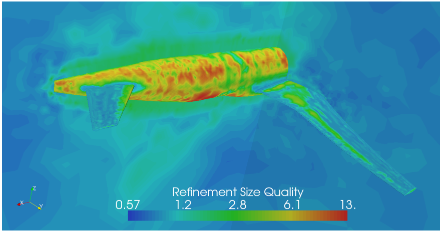


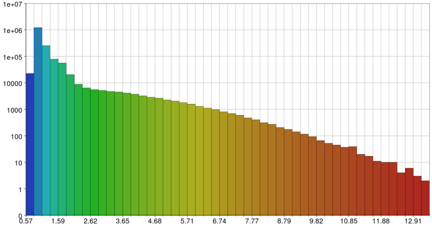


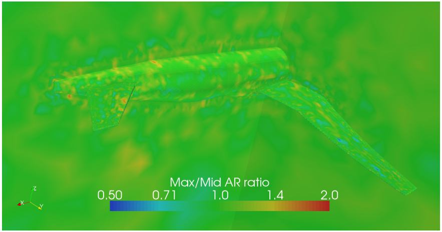


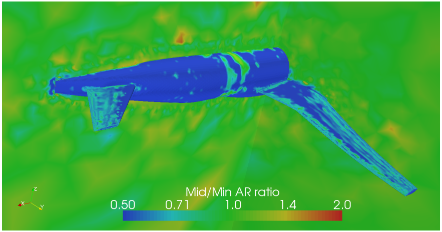


Langley CC Medium-Fine Comparison

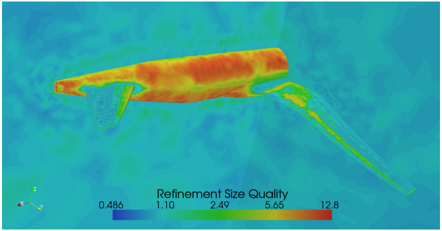


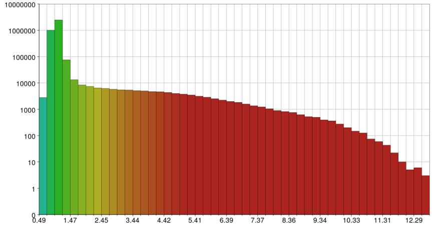


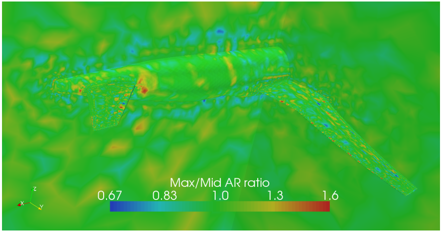




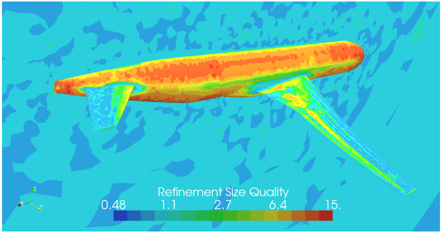
Langley VC Coarse-Medium Comparison

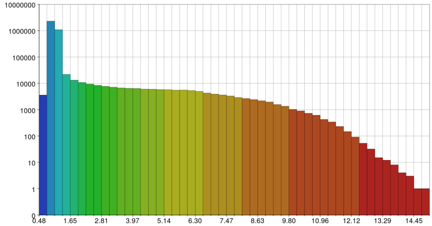


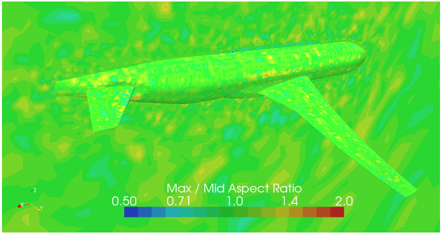




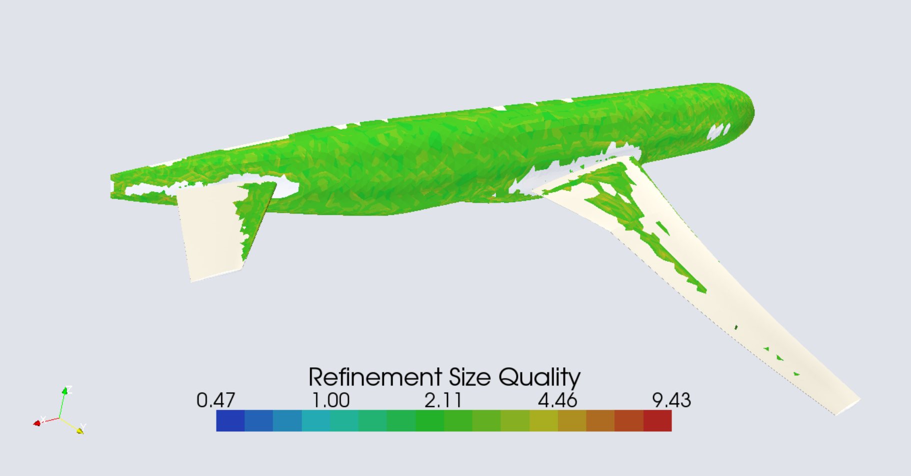
Cessna Coarse-Medium Comparison



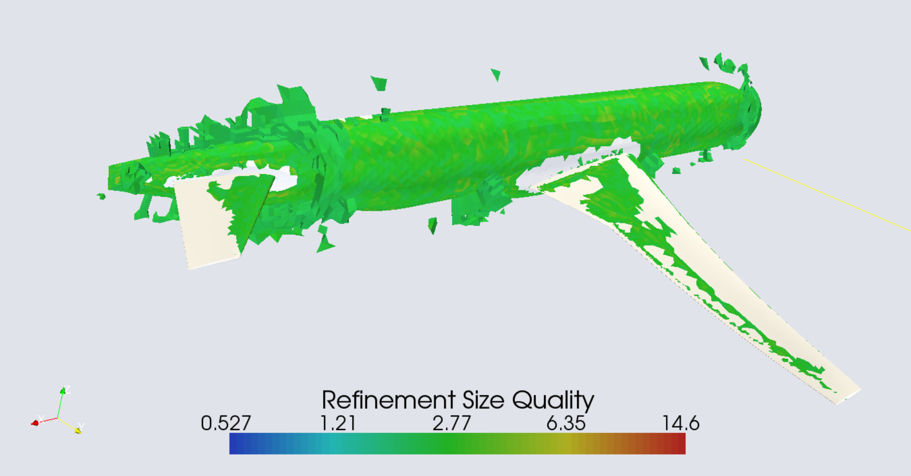




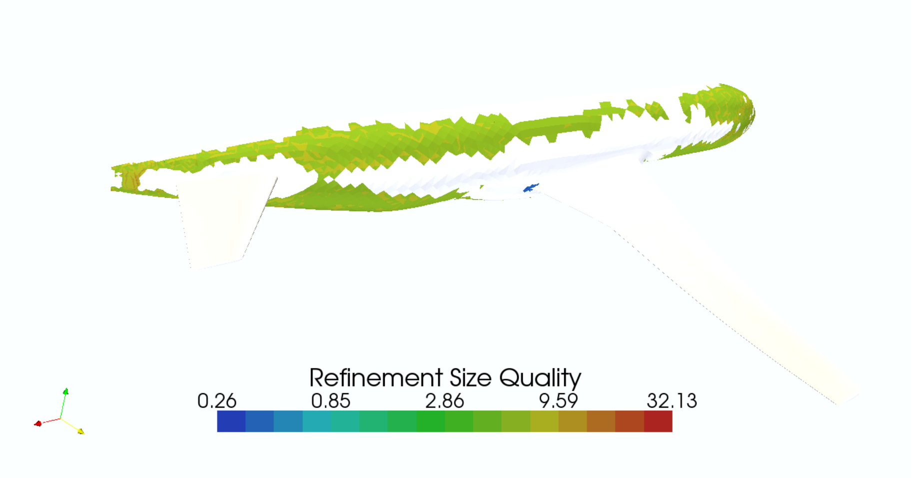
Langley CC-Langley CC Size Comparison



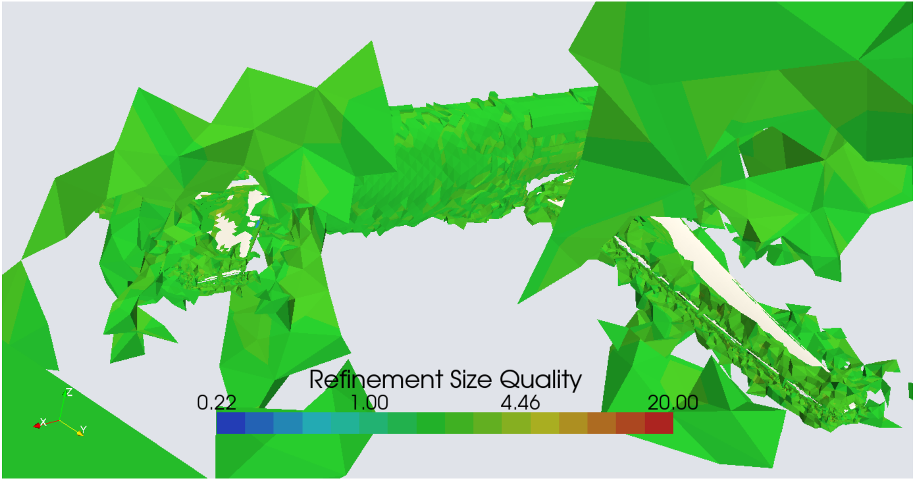
Langley CC-Langley VC Size Comparison



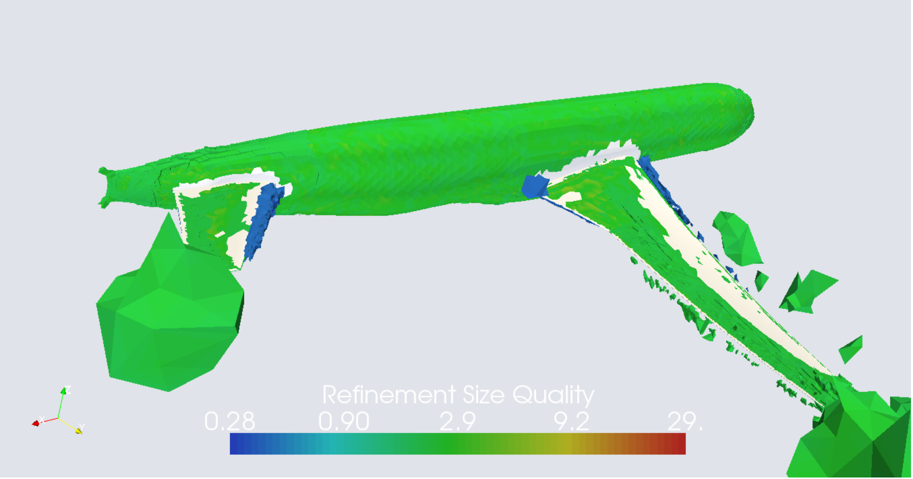
Langley CC-Cessna Size Comparison



Langley CC-Boeing Size Comparison



Langley CC-Boeing Best Practice Size Comparison



Mesh Quality Measures

Current analysis tool

- Continue V&V, efficiency improvements
- More careful look at aspect ratio, shortest edge

Other mesh quality analysis

- Geometric cell quality measures — carefully!
- Cell-to-cell variation

Towards solution-based analysis

- Approximation error for manufactured / simplified solution

More Sophisticated measures

- Given both mesh *and* solution, could look at solution approximation error:
  - Mesh metric versus solution Hessian
  - Accuracy of reconstruction
- With mesh, solution, and solver code, could also look at local truncation error:
  - Flux integral for converged solution of different solver
    - Including different flux functions, turbulence models, etc
  - Flux integral for manufactured/simplified solution
- Ideally, all the data that comes out of this can guide mesh improvement and/or be used to test solver variations

How to Contribute Your Data

- Upload via anonymous ftp to mesh-quality.mech.ubc.ca
  - Path: pub/incoming/DPW4 (or pub/incoming for non-DPW data)
- CGNS data preferred but we want you data anyway!
- Solution data, plus mesh data (or tell us which DPW4 mesh you used)
  - Compressed archives (tar.gz, tar.bz2, zip, etc)
  - Include text file describing file format explicitly and send a code snippet that reads or writes the data to remove all ambiguity
- Unless otherwise requested, uploaded data will be put into outgoing directory for other researchers to work with as well
- Send email to cfog@mech.ubc.ca with questions / problems